The COMICS Tool – Computing Minimal Counterexamples for DTMCs

Nils Jansen[1], Erika Ábrahám[1], Matthias Volk[1], Ralf Wimmer[2], Joost-Pieter Katoen[1], and Bernd Becker[2]

[1] RWTH Aachen University, Germany
[2] Albert-Ludwigs-University Freiburg, Germany

Abstract. This paper presents the tool `COMICS` 1.0, which performs model checking and generates counterexamples for DTMCs. For an input DTMC, `COMICS` computes an abstract system that carries the model checking information and uses this result to compute a *critical subsystem*, which induces a counterexample. This abstract subsystem can be refined and concretized *hierarchically*. The tool comes with a command line version as well as a graphical user interface that allows the user to interactively influence the refinement process of the counterexample.

1 Introduction

Discrete-time Markov chains (DTMCs) are widely used to model safety-critical systems with uncertainties. Model checking *probabilistic computation tree logic* (PCTL) properties can be performed by prominent tools like Prism [1] and Mrmc [2]. Unfortunately, the implemented numerical methods do not provide diagnostic information in form of *counterexamples*, which are very important for debugging and are also needed for CEGAR frameworks [3].

Although different approaches [4,5,6] were proposed for probabilistic counterexamples, there is still a lack of efficient and user-friendly *tools*. To fill this gap, we developed the tool `COMICS` (Computing Minimal Counterexamples), , supporting SCC-based model checking [7] and, in case the property is violated, the *automatic* generation of *abstract counterexamples* [5], which can be subsequently refined either automatically or user-guided.

While most approaches represent probabilistic counterexamples as sets of paths, we use (hierarchically abstracted) subgraphs of the input DTMC, so-called *critical subsystems*. This allows for a much more *compact representation* and a *significant decrease in the computational complexity*. The user can refine abstract critical subsystems *hierarchically* by choosing system parts of interest which are to be concretized and further examined. All computation steps of the hierarchical counterexample refinement can be *guided and revised*. Though refinement can be done until a fully concrete counterexample is gained, it seems likely that the user can gain sufficient debugging information from abstract systems considering real-world examples with millions of states. The tool's graphical user interface (GUI) permits *visualization, reviewing and creation* of test cases.

The only other available tool we are aware of is DiPro [8], which supports both DTMCs and CTMCs but no abstract counterexamples, which is crucial for the handling of large systems. It also does not allow the user to influence the search by using his or her expertise. Comparative experiments show that we can compute reasonably smaller counterexamples in shorter time with our tool.

In Section 2 we recall some foundations andgive a brief introduction to the methods implemented in our tool. We describe the features and architecture and report on benchmarks in Section 3. We conclude the paper in Section 4. The tool, a detailed manual, and a number of benchmarks are available at the `COMICS` website[3].

2 Foundations

In this section we give some basic foundations and briefly explain the algorithms implemented in `COMICS` (see [5] and [9] for more details).

Definition 1. *Assume a set AP of atomic propositions. A* discrete-time Markov chain (DTMC) *is a tuple $M = (S, I, P, L)$ with a non-empty finite state set S, an initial discrete probability distribution $I : S \to [0,1]$ with $\sum_{s \in S} I(s) = 1$, a transition probability matrix $P : S \times S \to [0,1]$ with $\sum_{s' \in S} P(s,s') = 1$ for all $s \in S$, and a labeling function $L : S \to 2^{AP}$.*

Assume in the following a set AP of atomic propositions and a DTMC $M = (S, I, P, L)$.

We say that there is a *transition* from a state $s \in S$ to a state $s' \in S$ iff $P(s,s') > 0$. A *path* of M is a finite or infinite sequence $\pi = s_0 s_1 \ldots$ of states $s_i \in S$ such that $P(s_i, s_{i+1}) > 0$ for all i. We say that the transitions (s_i, s_{i+1}) are *contained* in the path π, written $(s_i, s_{i+1}) \in \pi$. We write $Paths^M_{inf}$ for the set of all infinite paths of M, and $Paths^M_{inf}(s)$ for those starting in $s \in S$. Analogously, $Paths^M_{fin}$ is the set of all finite paths of M, $Paths^M_{fin}(s)$ of those starting in s, and $Paths^M_{fin}(s,t)$ of those starting in s and ending in t. A state t is called *reachable* from another state s iff $Paths^M_{fin}(s,t) \neq \emptyset$.

A state set $S' \subseteq S$ is called *absorbing in M* iff there is a state in S' from which no state outside S' is reachable in M. We call S' *bottom in M* if this holds for all states in S'. States $s \in S$ with $P(s,s) = 1$ are also called *absorbing states*.

We call M *loop-free*, if all of its loops are self-loops on absorbing states. A set $S' \subseteq S$ is *strongly connected in M* iff for all $s, t \in S'$ there is a path from s to t visiting states from S' only. A *strongly connected component (SCC)* of M is a maximal strongly connected subset of S.

The probability measure for finite paths $\pi \in Paths^M_{fin}$ is defined by $Pr^M_{fin}(\pi) = \prod_{(s_i, s_{i+1}) \in \pi} P(s_i, s_{i+1})$. For a set $R \subseteq Paths^M_{fin}$ of paths we have $Pr^M_{fin}(R) = \sum_{\pi \in R'} Pr^M_{fin}(\pi)$ with $R' = \{\pi \in R \mid \forall \pi' \in R.\ \pi' \text{ is no prefix of } \pi\}$.

The syntax of *probabilistic computation tree logic (PCTL)* [10] is given by[4]

$$\varphi \ ::= \ p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathbb{P}_{\sim\lambda}(\varphi \ U \ \varphi)$$

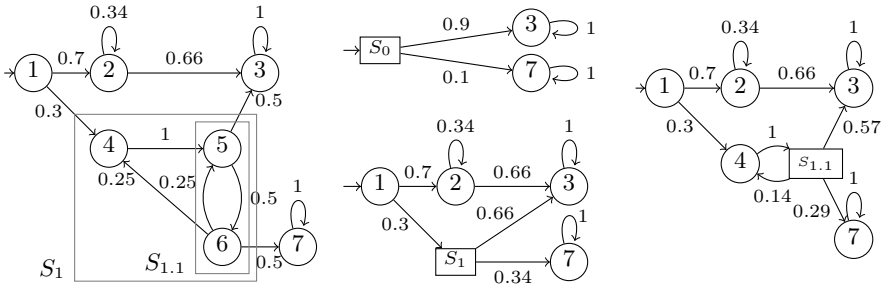
[3] `http://www-i2.informatik.rwth-aachen.de/i2/comics/`

[4] In this paper we only consider unbounded properties.

for (state) formulae with $p \in AP$, $\lambda \in [0,1] \subseteq \mathbb{R}$, and $\sim \in \{<, \leq, \geq, >\}$. We define the "next"-operator ($\Diamond$) and the "globally"-operator $\Box$ in the usual way.

For a property $\mathbb{P}_{\leq\lambda}(\varphi_1 \, U \, \varphi_2)$ refuted by M, a *counterexample* is a set $C \subseteq Paths^M_{fin}$, $Pr^M_{fin}(C) > \lambda$ of finite paths starting in an initial state and *satisfying* $\varphi_1 \, U \, \varphi_2$. For $\mathbb{P}_{<\lambda}(\varphi_1 \, U \, \varphi_2)$, the probability mass has to be at least λ. We consider upper probability bounds; see [4] for the reduction of lower bounds to this case. The φ-states are also called *target* states. We concentrate on this case and assume DTMCs to have single initial and target states. Note, that each DTMC can be equivalently transformed to satisfy these requirements.

In [7] we proposed a model checking approach for DTMCs based on *hierarchical abstraction*. Each SCC of the underlying graph of the input DTMC is abstracted by a state whose outgoing transitions lead to states outside the SCC and carry the whole probability mass of reaching those states when once entering the SCC. This abstraction is done recursively in a bottom-up manner: before abstracting an SCC we first apply abstraction to the sub-SCCs nested in it. The final result is an abstract DTMC whose only transitions lead from the initial state of the input DTMC to absorbing states and carry the corresponding reachability probabilities. Fig. 1(a) depicts a DTMC and its nested SCC structure: SCC S_1 contains SCC $S_{1.1}$. The upper graph of Fig. 1(b) depicts the result of the model checking: The probability to reach the target state 3 from the initial state 0 is 0.9. This abstract DTMC can be also concretized hierarchically. The lower graph of Fig. 1(b) shows the concretization of the abstract state S_0: The outgoing edges of S_1 carry the probability mass of all paths leading from the input state 4 of S_1 to the output states 3 and 7. Fig. 1(c) shows a further concretization of S_1 while SCC $S_{1.1}$ is still abstracted. Concretizing also $S_{1.1}$ would result in the DTMC of Fig. 1(a). Based on this approach, in [5] we presented a method to compute and represent counterexamples as *critical subsystems*, consisting of subsets of the original DTMC's states and transitions such that the probability of reaching target states from the initial state within the subsystem still exceeds the probability bound λ. The method first computes an abstract critical subsystem for the abstract DTMC resulting from model checking. Inside this abstract DTMC one or more abstract states are selected and concretized, and a critical subsystem is determined for the concretized system. This process may be repeated until the system is fully concretized. We suggested two methods for the computation of critical subsystems: The *global search* (`GS`) looks for for most probable paths through the whole system until the involved states and transitions form a critical subsystem. The *local search* (`LS`) builds critical subsystems incrementally by extending subsystems with most probable path fragments. Its application to benchmarks showed the competitiveness of the SCC-based model checking. Compared to other approaches, experiments for the counterexample generation revealed an improvement by several orders of magnitude in the number of paths needed to form the counterexample as well as in the number of involved states.



(a) Input DTMC

(b) Model checking result and concretization of S_0

(c) Concretization of S_1

Fig. 1. Example SCC-based model checking

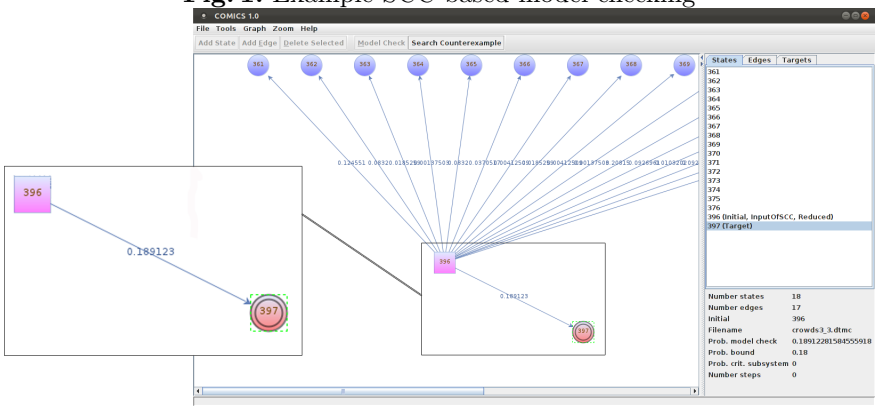


Fig. 3. Screenshot of COMICS's GUI with an instance of the crowds protocol

3 The COMICS Tool

COMICS can be used either as a command-line tool or with a GUI, the latter allowing the user to actively influence the process of finding a counterexample. The program consists of approximately 20 000 lines of code in five main components (see Fig. 2). The GUI is implemented in Java, all other components in C++. The user may select *exact* or *floating point* arithmetics for the computations.

SccMC performs *model checking* for an input DTMC and returns an abstract DTMC to Concretize or to GUI. Concretize *selects and concretizes* some states, either automatically or user-guided via the GUI. CritSubSys can be invoked on the modified system to compute a critical subsystem using GS or LS.

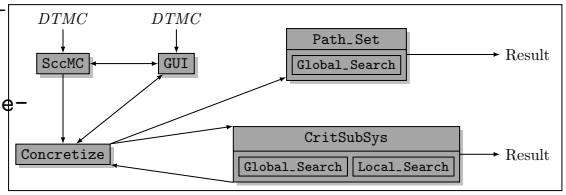


Fig. 2. Architecture of COMICS

		crowds						contract signing			
states		3515		18817	198199	485941	1058353	33790	156670	737278	1654782
transitions		6035		32677	198199	857221	1872313	34813	157693	753663	1671165
total prob.		0.2346		0.4270	0.7173	0.809	0.8731	0.5156	0.5156	0.5039	0.5039
prob. threshold		0.15	0.23	0.25	0.35	0.4	0.4	0.5	0.5	0.5	0.5
GS	# states	629	1071	2036	5198	5248	5250	6827	37601	140034	369448
	prob.	0.1501	0.2301	0.25	0.3503	0.4002	0.4001	0.5	0.5	0.5	0.5
	time (s)	0.02	0.38	0.38	7.97	16.36	18.78	0.36	2.98	238.82	605.81
LS	# states	182	900	943	4180	6368		6657	37377		
	prob.	0.1501	0.2302	0.2501	0.3501	0.4	TO	0.5	0.5	MO	MO
	time (s)	0.14	1.11	6.1	619.06	2455.46		8	54.58		
kSP	# states	1071						6827	37601	140034	369444
	prob.	0.15	TO	TO	TO	TO	TO	0.5	0.5	0.5	0.5
	time (s)	6.58						1.93	0.13	0.69	1.49
DiPro	# states	938	2901	3227	9005			13311	74751		
	prob.	0.1675	0.2334	0.254	0.3533	ERR	ERR	0.5	0.5	MO	MO
	time (s)	2.02	7.06	7.87	44.34			1210	7114		

Fig. 4. Results for crowds and contract signing (TO $> 2h$)

The result is given back to `Concretize` for further refinement or returned as the result. *Heuristics* for the number of states to concretize in a single step as well as for the choice of states are offered. It is also possible to predefine the number of concretization steps. Counterexample representations as *sets of paths* and as *critical subsystems* are offered. The first case yields a *minimal counterexample* [4]. The `GUI` provides a *graph editor* for specifying and modifying DTMCs. A large number of *layout algorithms* increase the usability even for large graphs. Both concrete and abstract graphs can be *stored*, *loaded*, *abstracted*, and *concretized* by the user. As the most important feature, the user is able to *control the hierarchical concretization* of a counterexample. If an input graph seems too large to display, the tool offers to operate without the graphical representation. In this case the abstract graph can be computed and refined in order to reduce the size. Fig. 3 shows one abstracted instance of the *crowds protocol* benchmark [11], where the probability of reaching the unique target state is displayed in the information panel on the right as well as on the edge leading from the initial state to the target state. The initial state is abstract and can therefore be expanded.

Fig. 3 provides a comparison with DiPro [8]. We applied our tool using `GS`, `LS` and the k-shortest path (k`SP`) approach [4] to the *crowds protocol* and the *probabilistic contract signing protocol* [12] for different probability thresholds all smaller than the model checking result (total prob.). We measured the size of the counterexample (states), the probability of reaching target states (prob.) and the computation time excluding the initial model checking. TO denotes timeout, MO out of memory and ERR wrong result. On the crowds protocol, `GS` performs best, while `LS` computes in general smaller counterexamples. k`SP` is the fastest method for contract signing, however, the representation of the result consists of a huge number of paths instead of a small subsystem of the input DTMC.

4 Conclusion and Future Work

We presented version 1.0 of our tool `COMICS` which generates abstract, hierarchically refinable counterexamples for DTMCs. In the future, we will integrate the computation of *minimal* critical subsystems [6] and the adaption of our approaches to *symbolic data structures*. We are also working on an *incremental version of the*

Dijkstra algorithm for path search and on *compositional counterexamples*, since PRISM models are usually built by parallel composition.

Acknowledgments We thank our student researchers Jens Katelaan, Maik Scheffler, Andreas Vorpahl, and Barna Zajzon for their excellent work on the project.

References

1. Kwiatkowska, M.Z., Norman, G., Parker, D.: PRISM 4.0: Verification of probabilistic real-time systems. In: Proc. of CAV. Volume 6806 of LNCS, Springer (2011) 585–591
2. Katoen, J.P., Zapreev, I.S., Hahn, E.M., Hermanns, H., Jansen, D.N.: The ins and outs of the probabilistic model checker MRMC. Perform. Eval. **68**(2) (2011) 90–104
3. Hermanns, H., Wachter, B., Zhang, L.: Probabilistic CEGAR. In: Proc. of CAV. Volume 5123 of LNCS, Springer (2008) 162–175
4. Han, T., Katoen, J.P., Damman, B.: Counterexample generation in probabilistic model checking. IEEE Trans. on Software Engineering **35**(2) (2009) 241–257
5. Jansen, N., Ábrahám, E., Katelaan, J., Wimmer, R., Katoen, J.P., Becker, B.: Hierarchical counterexamples for discrete-time Markov chains. In: Proc. of ATVA. Volume 6996 of LNCS, Springer (2011) 443–452
6. Wimmer, R., Jansen, N., Ábrahám, E., Becker, B., Katoen, J.P.: Minimal critical subsystems for discrete-time Markov models. In: Proc. of TACAS. LNCS, Springer (2012)
7. Ábrahám, E., Jansen, N., Wimmer, R., Katoen, J.P., Becker, B.: DTMC model checking by SCC reduction. In: Proc. of QEST, IEEE CS (2010) 37–46
8. Aljazzar, H., Leitner-Fischer, F., Leue, S., Simeonov, D.: DiPro – A tool for probabilistic counterexample generation. In: Proc. of SPIN. Volume 6823 of LNCS, Springer (2011) 183–187
9. Jansen, N., Ábrahám, E., Katelaan, J., Wimmer, R., Katoen, J.P., Becker, B.: Hierarchical counterexamples for discrete-time Markov chains. Technical Report AIB-2011-11, RWTH Aachen University (2011)
10. Hansson, H., Jonsson, B.: A logic for reasoning about time and reliability. Formal Aspects of Computing **6**(5) (1994) 512–535
11. Reiter, M.K., Rubin, A.D.: Crowds: Anonymity for web transactions. ACM Trans. on Information and System Security **1**(1) (1998) 66–92
12. Norman, G., Shmatikov, V.: Analysis of probabilistic contract signing. Journal of Computer Security **14**(6) (2006) 561–589