Quantum Processes and Computation

Assignment 1, Wednesday, January 30th, 2019

Exercise teachers:
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Tuesday, February 5th, 12:00

Goals: After completing these exercises successfully you should be able to perform simple diagrammatic computations. The total number of points is 100, distributed over 6 exercises.

Exercise 1 (3.4) (20 points): We saw in the lecture that **functions** and **relations** are examples of process theories. Give two other examples of a process theory. For each one answer the following questions:

1. What are the system-types?
2. What are the processes?
3. How do processes compose, both sequentially and parallel?
4. When should two processes be considered equal?

Hint: Be creative! You don't have to restrict yourself to mathematics.

Exercise 2 (3.10) (20 points): Please read Section 3.1.3 about diagrams as diagram formulas. Draw the diagrams corresponding to the following diagram formulas:

1. $f_{B_1C_2}^{C_4} g_{C_4}^{D_3}$
2. $f_{A_1}^{A_1}$
3. $g_{B_1}^{A_1} f_{A_1}^{B_1}$
4. $1_{A_1}^{A_6} 1_{A_2}^{A_5} 1_{A_3}^{A_4}$.

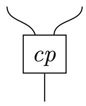
Use the convention that inputs and outputs are numbered from left-to-right.

Exercise 3 (3.12) (20 points): Give the diagrammatic equations of a process $*$ taking two inputs and one output that express the algebraic properties of being

1. associative: $x * (y * z) = (x * y) * z$
2. commutative: $x * y = y * x$
3. having a unit: there exists a process e (with no inputs) such that $x * e = e * x = x$

Note: x, y and z should not appear in your final diagrams. They are however useful in trying to figure out what the diagrammatic equation should be.

Exercise 4 (3.15) (10 points): Using the copy operation:

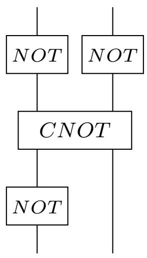


$$:: \quad n \mapsto (n, n)$$

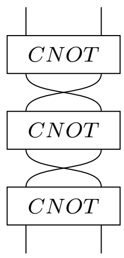
write down the diagram representing distributivity: $(x + y) * z = (x * z) + (y * z))$? Here, $+$ and $*$ are processes that take two inputs and and one output.

Exercise 5 (3.30) (10 points): First compute the values of the following functions, then give the commonly used name of these functions:

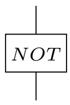
(a) :



(b) :

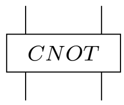


where:



$$:: \begin{cases} 0 \mapsto 1 \\ 1 \mapsto 0 \end{cases}$$

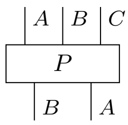
and



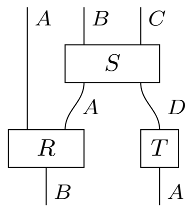
$$:: \begin{cases} (0,0) \mapsto (0,0) \\ (0,1) \mapsto (0,1) \\ (1,0) \mapsto (1,1) \\ (1,1) \mapsto (1,0) \end{cases}$$

Exercise 6 (3.31) (20 points): Suppose A, B, C, and D are sets and P is a relation given by:

$$\begin{aligned} A &= \{a_1, a_2, a_3\} \\ B &= \mathbb{B} \\ C &= \{\textbf{red}, \textbf{green}\} \\ D &= \mathbb{N} \end{aligned}$$



$:=$



Compute P first for R, S, T given by:

$$R :: \begin{cases} 1 \mapsto (a_1, a_1) \\ 1 \mapsto (a_1, a_2) \end{cases} \qquad S :: \begin{cases} (a_1, 5) \mapsto (0, \textbf{red}) \\ (a_1, 5) \mapsto (1, \textbf{red}) \\ (a_2, 6) \mapsto (1, \textbf{green}) \end{cases} \qquad T :: \begin{cases} a_1 \mapsto 200 \\ a_3 \mapsto 5 \end{cases}$$

and then for R, S, T given by:

$$R :: \begin{cases} 0 \mapsto A \times \{a_2, a_3\} \\ 1 \mapsto A \times \{a_2, a_3\} \end{cases} \qquad S :: \begin{cases} (a_1, 0) \mapsto \mathbb{B} \times \{\textbf{red}, \textbf{green}\} \\ (a_1, 1) \mapsto \mathbb{B} \times \{\textbf{red}, \textbf{green}\} \\ (a_1, 2) \mapsto \mathbb{B} \times \{\textbf{red}, \textbf{green}\} \\ \vdots \end{cases} \qquad T :: \begin{cases} a_1 \mapsto \mathbb{N} \\ a_2 \mapsto \mathbb{N} \\ a_3 \mapsto \mathbb{N} \end{cases}$$

Hint: Read Section 3.3.3.