Quantum Processes and Computation

Assignment 10, Wednesday, April 24, 2019

Exercise teachers:
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Handing in your answers: There are two options:

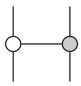
1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Tuesday, May 7, 12:00

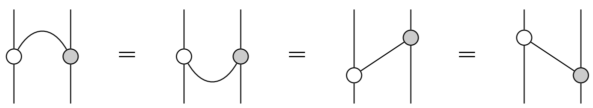
Goals: After completing these exercises you can reason with (strong) complementarity and can do concrete calculations with ZX-diagrams. The total number of points is 100, distributed over 3 exercises.
Material covered in book: sections 9.2, 9.3, 9.4.

Note: In this exercise sheet ○ and ● will always represent strongly complementary spiders.

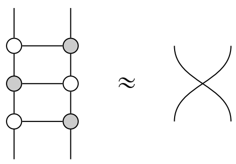
Exercise 1 (9.47) (30 points): Read Section 9.2.3 about the controlled-NOT gate (CNOT):



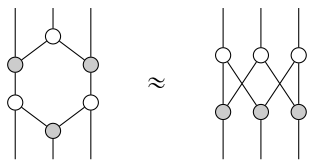
(i) Complete Lemma 9.46 by proving the remaining equalities:



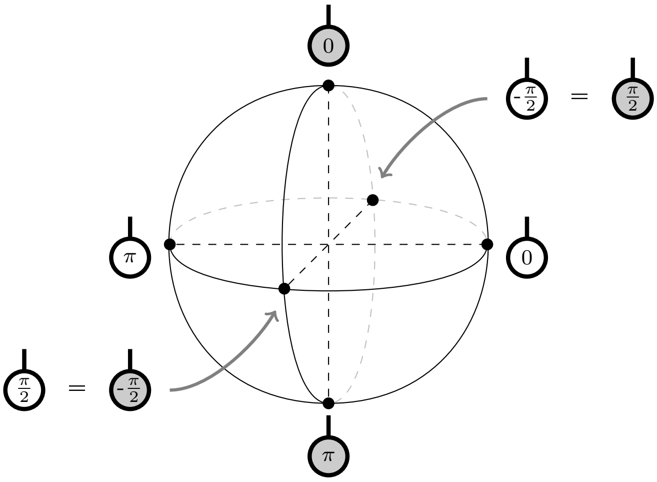
(ii) Use complementarity and strong complementarity to prove that



and



The ZX-calculus is based on the Z- and X-spiders and bases, but of course the Bloch sphere has a third axis: the Y-axis. The 'Y-basis' states can be represented in two different ways as Z- and X-phase spiders:

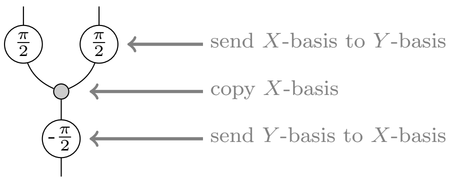


But in fact, these expressions are only equal because we have doubled the states.

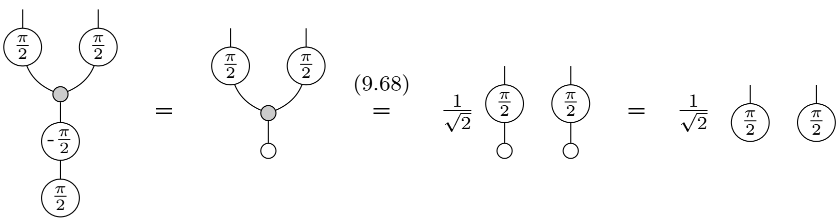
Exercise 2 (9.106) (30 points): Using the concrete definitions of the Z- and X-spider, show that

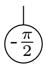
 (1)

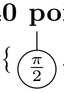
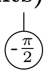
The two ways of writing the Y-basis states also allow us to find two different ways to copy these states. The first is:

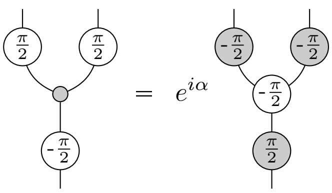


because



and similarly with  (see the text above Exercise 9.107 in the book).

Exercise 3 (9.107) (40 points): Using the equalities derived in the previous exercise, and by exploiting the fact that {, } forms a basis for $\mathbb{C}^2$, show that



for some fixed global phase $e^{i\alpha}$.

We will refer to this equality as the *Y-rule* in the future, and it will be important for us in the next lecture, as it allows us to change the colours of a spider in a diagram.