# Quantum Processes and Computation

**Assignment 11, Wednesday, May 8, 2019**

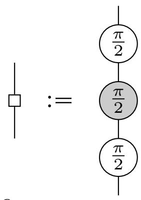
**Exercise teachers:**
Aleks Kissinger (`aleks@cs.ru.nl`)
John van de Wetering (`wetering@cs.ru.nl`)

**Handing in your answers:** There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

**Deadline:** Tuesday, May 14, 12:00

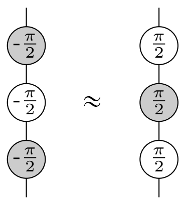
**Goals:** After doing these exercises you know about a large class of quantum gates and you can reason with them. The total number of points is 100, distributed over 3 exercises.
Material covered in book: sections 9.4, 12.1.

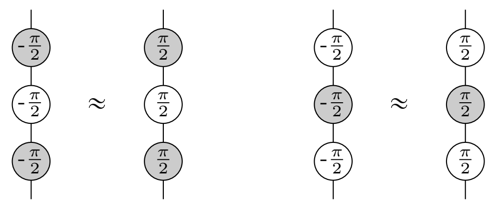
The Hadamard gate was defined as 

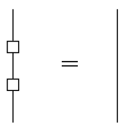
There are a few other equivalent definitions.

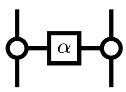
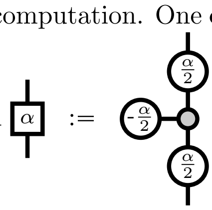
**Exercise 1 (9.112) (40 points):**

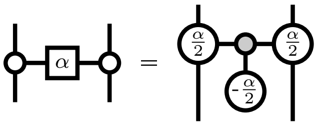
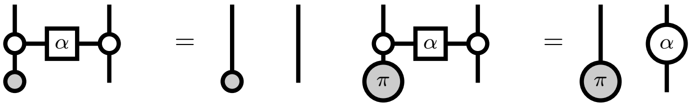
(i) Follow the instructions of Exercise 9.112 in the book to show that:



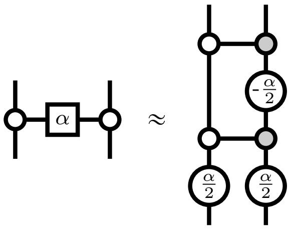
(ii) Show that: 

Hint: Use that  and that inverses of maps are unique.

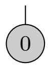
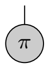
There are a couple of sets quantum gates that crop up a lot in quantum computation. One of these is the *controlled phase* gate CZ($\alpha$):  where $\alpha$ is any phase and  so that

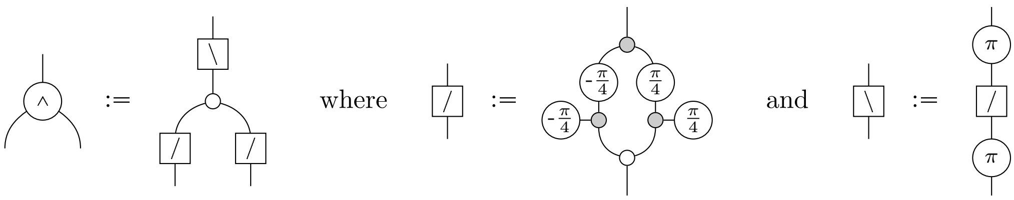
 and indeed 

**Exercise 2 (12.8) (20 points):** Show using the ZX-calculus that the CZ($\alpha$)-gate can be built from CNOTs and ○ phase gates as follows:

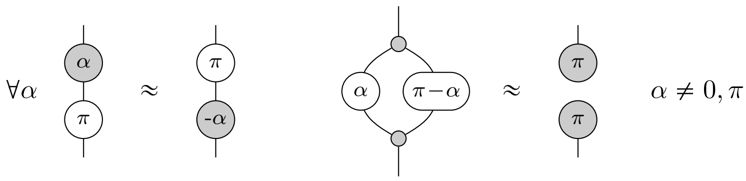


Recall that a single qubit unitary can be written as a succession of phase gates using a Euler Decomposition of the unitary. Since we have controlled phase gates in the ZX-calculus, we therefore have controlled single qubit unitaries. CNOT is a 2-qubit gate. Can we also make a controlled version of it? This would then be a 'controlled controlled not' gate, more commonly known as a *Toffoli*-gate. This is a three qubit gate that performs a NOT operation on the third qubit if the first two are in the $|1\rangle$ state. This process of course involves an AND-gate. Constructing this gate is surprisingly involved.

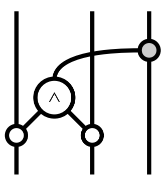
**Exercise 3 (12.10) (40 points):** By evaluating on the Z-basis states  and  show that the $\wedge$ map below acts as a AND-gate.



You will need to use the following rules (that we haven't proven) and their colour reverses:



With the AND-gate in hand we can define the Toffoli-gate:



The Toffoli-gate is a very powerful gate. As is shown in Section 12.1.3 of the book, any unitary can be implemented in the ZX-calculus with the help of the Toffoli-gate. Somewhat surprisingly, it is the only gate you need for doing reversible classical computation, and approximate universal quantum computation can be achieved using just Toffoli- and Hadamard-gates.