Quantum Processes and Computation

Assignment 3, Wednesday, February 13, 2019

Exercise teachers:
Aleks Kissinger (`aleks@cs.ru.nl`)
John van de Wetering (`wetering@cs.ru.nl`)

Handing in your answers: There are two options:

1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

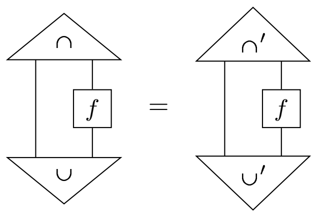
Deadline: Tuesday, February 19, 12:00

Goals: After completing these exercises you know how to reason with the transpose, adjoints, and the conjugate. You know about projections, unitaries and isometries. The total number of points is 100, distributed over 5 exercises.
Material covered in book: Chapter 4

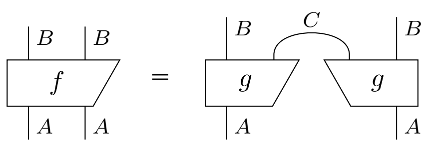
Exercise 1 (4.59) (15 points): An *inverse* for a process $f : A \to B$ is a process $f^{-1} : B \to A$ such that $f^{-1} \circ f = \mathrm{id}_A$ and $f \circ f^{-1} = \mathrm{id}_B$. Show that for a process f the following are equivalent:

- f is unitary.
- f is an isometry and has an inverse.
- $f^\dagger$ is an isometry and has an inverse.

Exercise 2 (4.37) (20 points): Show that the trace of a process is independent of the particular choice of cup and cap, i.e. that if $\cup$ and $\cap$ satisfy the yanking equations, but $\cup'$ and $\cap'$ also satisfy it that then:



In the lecture we saw the notion of a positive process. There is also a notion of $\otimes$-positivity. A process f is $\otimes$-positive if there exists a process g such that

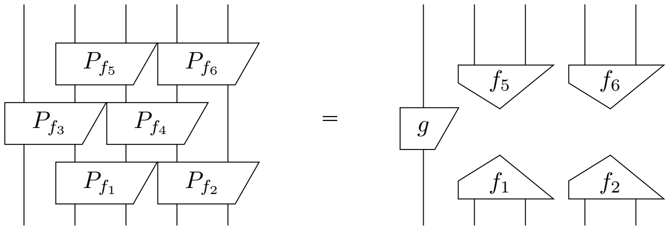


(see section 4.3.6 of the book for more information)

Exercise 3 (4.67) (15 points): Show that the sequential composition of two $\otimes$-positive processes is again a $\otimes$-positive process.

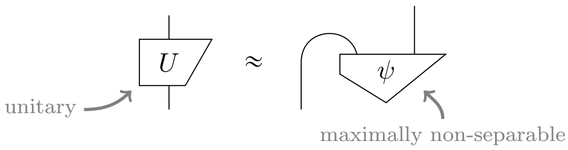
For a process $f : A \to A$ we define its *separable projector* by

Exercise 4 (4.73) (30 points): Given processes $f_i : A \to A$ find the process g such that:

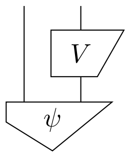


Write g as a sequential composition of the conjugates, transposes and adjoints of the f_i's.
Hint: Doing exercise 4.73 from the book first might reveal whether you understand the concept.

A state ψ is *maximally non-separable* if it corresponds to a unitary U by process-state duality, up to a number:



Exercise 5 (4.82) (20 points): Show (i) that if one applies a unitary V to one side of a maximally non-separable state:



that one again obtains a maximally non-separable state, and (ii) that this unitary can always be chosen such that the resulting state is the cup (up to a number).