Quantum Processes and Computation

Assignment 5, Wednesday, February 27, 2019

Exercise teachers:
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Handing in your answers: There are two options:

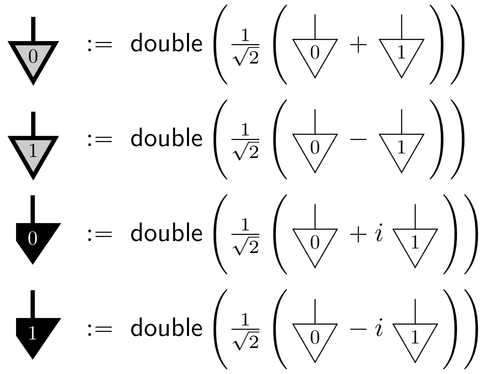
1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Tuesday, March 5, 12:00

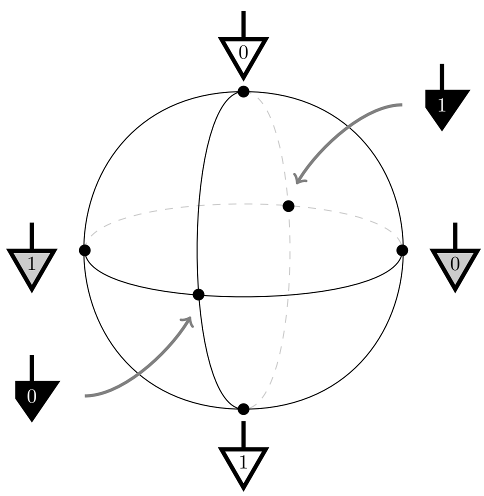
Goals: After completing these exercises you will know how to plot quantum states on the Bloch sphere and gain an understanding of the relationship between the process theories of **linear maps** and **quantum maps**. The total number of points is 100, distributed over 3 exercises.
Material covered in book: sections 5.3.3 and 6.1.

In the lecture (and in section 6.1.2), it was show that 2D quantum pure states correspond to points on a sphere.

Exercise 1 (6.7) (30 points): Show that the following points:



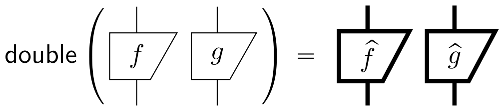
are located on the Bloch sphere as follows:



Hint: You may want to check out the *polar form* of a complex number, if you've never seen it before. It's covered in section 5.3.1.

Exercise 2 (6.10 & 6.22) (40 points):

(i) Show that doubling preserves parallel composition:

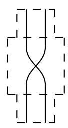


(ii) Show that doubling preserves normalisation: that a state ψ is normalised if and only if its doubled state $\widehat{\psi}$ is normalised.

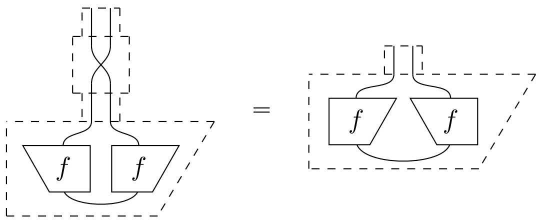
(iii) Show that doubling preserves orthogonality: that states ψ and ϕ are orthogonal if and only if $\widehat{\psi}$ and $\widehat{\phi}$ are orthogonal.

Hint: Use theorem 6.17 for the latter two points.

The transpose of a positive process is again a positive process and by bending some wires we can also take the 'transpose' of a $\otimes$-positive state, i.e. of a quantum state (see **Corollary 6.36**). This transpose acts as a swap of wires on the doubled system:

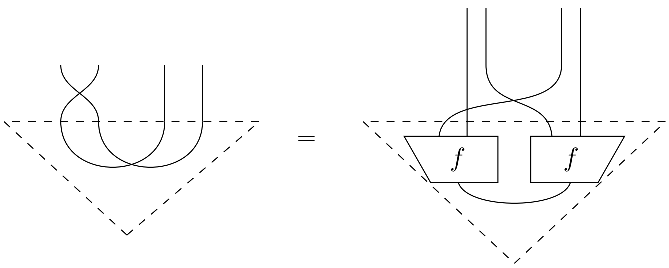


and it indeed sends quantum states to quantum states:



In the next exercise we will show that nevertheless, this swap of wires is *not* a quantum operation.

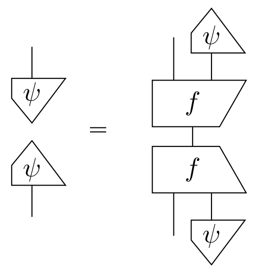
Exercise 3 (30 points): In this exercise we will show that a swap applied to one pair of the wires of the doubled cup state will result in a state that is no longer $\otimes$-positive, and therefore not a quantum state. We will do this by contradiction. So suppose:



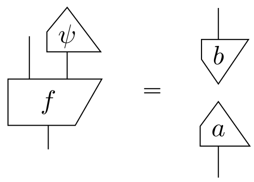
(1)

for some process f.

(i) Let ψ be a normalised state. Show that the equation above implies that



and hence, by Proposition 5.74, that there exist states a and b such that:



(2)

(ii) Plug ψ into equation 1 and use equation 2 to show that the identity wire disconnects. Conclude that therefore the swap can't be a quantum map.

Note: In proposition 6.48 it is also shown that the swap is not a quantum operation, but it uses a specific counter-example found in **linear maps**. The proof above only uses string diagrams and the property implied by proposition 5.74.