Quantum Processes and Computation

Assignment 8, Wednesday, April 10, 2019

Exercise teachers:
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Handing in your answers: There are two options:

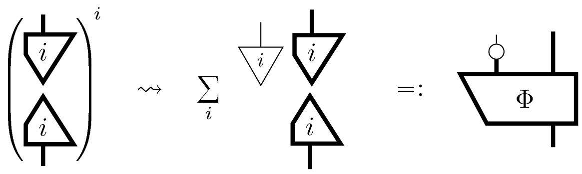
1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Tuesday, April 16, 12:00

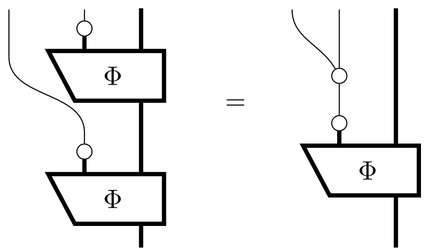
Goals: After completing these exercises you can work with classical-quantum maps and you can reason with classical spiders. The total number of points is 100, distributed over 4 exercises. Material covered in book: sections 8.1 and 8.2.

In last week exercises we saw that if we perform an ONB measurement twice, that we will always get the same result the second time. Now that we have classical-quantum maps we can represent this diagrammatically.

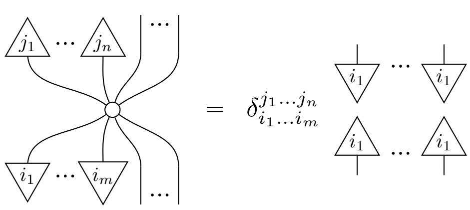
Exercise 1 (20 points): Write a non-demolition ONB-measurement in the classical-quantum notation:



and show that



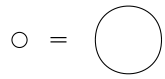
Exercise 2 (8.32) (10 points): Prove the *generalised copy rule* for spiders:



where $\delta^{j_1 \dots j_n}_{i_1 \dots i_m}$ is the generalised Kronecker delta that is 1 when all the in- and outputs match and is 0 otherwise.

Exercise 3 (8.37 & 8.38) (40 points):

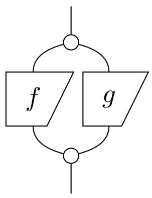
(i) Prove using just the spider fusion law that the spider with no legs equals the 'circle' (i.e. the dimension):



(ii) Not all spiders are causal. Determine which spiders are causal, which can be made causal by rescaling by a number, and which cannot.

Exercise 4 (8.26 & 8.55 & 8.69) (30 points):

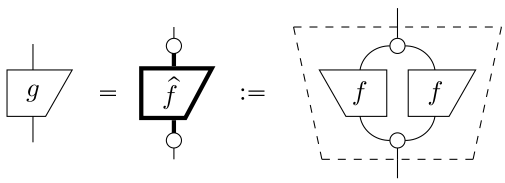
(i) Show that for any two linear maps f and g of the same type, the diagram:



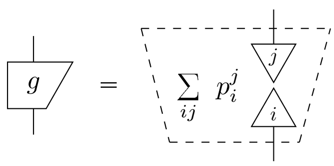
yields the Hadamard product of matrices:

$$\begin{pmatrix} f_1^1 & \cdots & f_D^1 \\ \vdots & \ddots & \vdots \\ f_1^D & \cdots & f_D^D \end{pmatrix} \star \begin{pmatrix} g_1^1 & \cdots & g_D^1 \\ \vdots & \ddots & \vdots \\ g_1^D & \cdots & g_D^D \end{pmatrix} = \begin{pmatrix} f_1^1 g_1^1 & \cdots & f_D^1 g_D^1 \\ \vdots & \ddots & \vdots \\ f_1^D g_1^D & \cdots & f_D^D g_D^D \end{pmatrix}$$

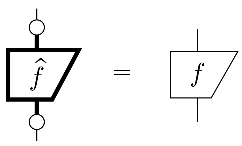
(ii) For a classical map g, find a pure quantum map $\widehat{f}$ such that



Hint: Use the representation



(iii) Show that a function map f (i.e. a deterministic causal classical map) satisfies



Hint: Use that classical maps are self-conjugate.