Quantum Processes and Computation

Assignment 9, Wednesday, April 17, 2019

Exercise teachers:
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Handing in your answers: There are two options:

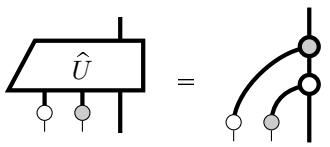
1. Deliver a hard copy to the mailbox of John van de Wetering. Mercator 1, 3rd floor.
2. E-mail a PDF to `wetering@cs.ru.nl`. Please include your name and the exercise number in the filename, e.g. `ACHTERNAAM-qpc-exercise1.pdf`.

Deadline: Tuesday, April 23, 12:00

Goals: After completing these exercises you can work with spiders of the bastard, quantum and phased kind. The total number of points is 100, distributed over 4 exercises.
Material covered in book: sections 8.3, 8.4, 9.1.

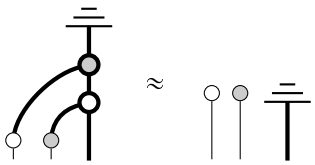
Exercise 1 (9.49, 9.58) (50 points)

1. Read Section 9.2.7 about teleportation with complementary spiders. To show that it is a valid protocol we must show that the actions of both Aleks and Bob can in fact be performed, i.e. that they are causal cq-maps. We already know that Aleks's side is causal, so it remains to show that Bob's side is as well, i.e. that

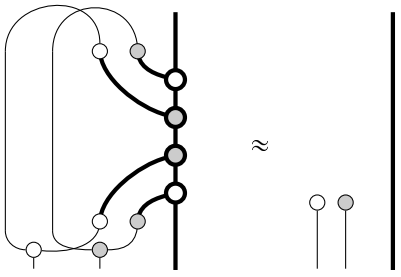


(1)

   is causal, up to a number:

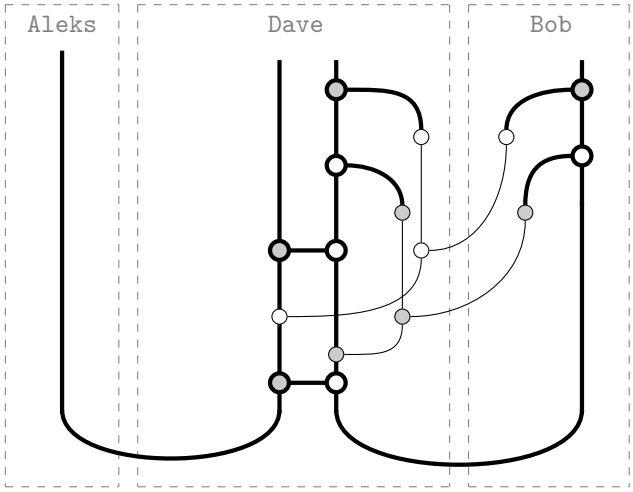


2. Show that furthermore, this map is a controlled isometry, up to a number, i.e.



In the future we might want to have a "quantum internet"[1] where users can do things using shared entangled state. However, the infrastructure needed for allowing any two users to directly create entangled states might be prohibitively expensive. The next exercise is about the protocol of *entanglement swapping*. In this protocol we have three parties, Aleks, Bob and Dave, where Aleks and Bob represent two users of our hypothetical quantum internet and Dave is a server. The protocol starts with the users Aleks and Bob sharing an entangled state with the server, Dave. We will show that Dave can perform some local quantum process and Bob some local correction so that in the end Aleks shares an entangled state with Bob. It is therefore sufficient for users to be able to create entanglement with a dedicated server, and perform local quantum operations in order to create a fully connected quantum internet.

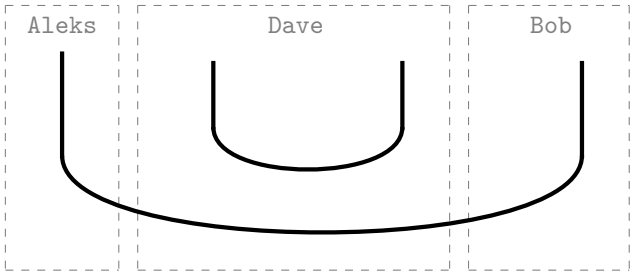
To be specific the protocol is:



(2)

So it consists of quantum CNOTs, measurements in ○ and ●, classical copies in ○ and ● and the controlled unitary (1).

Exercise 2 (9.60) (50 points): Show that (2) is equal to:



Hint: this is very similar to the proof that teleportation works given in section 9.2.7.

[1] That we will of course also want to connect to a centralised smart blockchain in the cloud.