Meyer and Wand property for Damas and Milner's polymorphic type assignment system

Presentation Type Theory and Coq

Jelte J. Zwetsloot

Radboud University Nijmegen

May 15th, 2018

Contents

Terms and values for call-by-name

untyped terms $e ::= x \mid \lambda x.e \mid e_1 e_2 \mid \textbf{let } x \textbf{ be } e_1 \textbf{ in } e_2$
$\textbf{callcc} \mid \textbf{throw}$

untyped values $n ::= \lambda x.e \mid \textbf{callcc} \mid \textbf{throw}$

Call-by-Name CPS Transform

$$
\begin{aligned}
|n|_{cbn} &= \lambda k.k||n||_{cbn} \\
|x|_{cbn} &= x \\
|e_1 e_2|_{cbn} &= \lambda k.|e_1|_{cbn}(\lambda k'.k'|e_2|_{cbn}k) \\
|\textbf{let } x \textbf{ be } e_1 \textbf{ in } e_2|_{cbn} &= \lambda k.\textbf{let } x \textbf{ be } |e_1|_{cbn} \textbf{ in } (|e_2|_{cbn}k)
\end{aligned}
$$

$$
\begin{aligned}
||\lambda x.e||_{cbn} &= \lambda x.|e|_{cbn} \\
||\textbf{callcc}||_{cbn} &= \lambda f.\lambda k.f(\lambda f'.f'(\lambda l.lk)k) \\
||\textbf{throw}||_{cbn} &= \lambda c.\lambda k.k(\lambda x.\lambda l.c(\lambda c'.x(\lambda x'.c'x')))
\end{aligned}
$$

DM types and contexts

mono-types	$\tau ::= t \mid b \mid \tau_1 \to \tau_2$
poly-types	$\sigma ::= \tau \mid \forall t.\sigma$
contexts	$\Gamma ::= \bullet \mid \Gamma, x : \sigma$

$\lambda^{\rightarrow}$ typing rules

$$\Gamma \vdash x : \Gamma(x) \ \text{(VAR)} \qquad \frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x.e : \tau_1 \rightarrow \tau_2} \ \text{(ABS)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2} \ \text{(APP)}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \textbf{let}\ x\ \textbf{be}\ e_1\ \textbf{in}\ e_2 : \tau_2} \ \text{(MONO-LET)}$$

Additional DM typing rules

$$\frac{\Gamma \vdash e : \sigma}{\Gamma \vdash e : \forall t.\sigma} \; (\text{t} \notin FTV(\Gamma)), (GEN) \qquad \frac{\Gamma \vdash e : \forall t.\sigma}{\Gamma \vdash e : [\tau/t]\sigma} \; (\text{INST})$$

$$\frac{\Gamma \vdash e_1 : \sigma \qquad \Gamma, x : \sigma \vdash e_2 : \tau}{\Gamma \vdash \textbf{let } x \textbf{ be } e_1 \textbf{ in } e_2 : \tau} \; (\text{x} \notin \text{dom}(\Gamma)), (POLY - LET)$$

DM+**cont** typing rules

DM is extended to $DM + \textbf{cont}$ by adding the type expression $\tau\,\text{cont}$, and by adding the following typing rules:

$$\Gamma \vdash \textbf{callcc} : \sigma_{callcc} \quad \text{(CALLCC')}$$

$$\Gamma \vdash \textbf{throw} : \sigma_{throw} \quad \text{(THROW')}$$

where $\sigma_{callcc} = \forall t.(t\ \text{cont} \rightarrow t) \rightarrow t$ and $\sigma_{throw} = \forall s.\forall t.s\ \text{cont} \rightarrow s \rightarrow t$.

Call-by-Name Type Transform for DM

$$
\begin{aligned}
|\tau|_{cbn} &= (||\tau||_{cbn} \to \alpha) \to \alpha \\
|\forall t.\sigma|_{cbn} &= \forall t.|\sigma|_{cbn}
\end{aligned}
$$

$$
\begin{aligned}
||t||_{cbn} &= t \\
||b||_{cbn} &= b \\
||\tau_1 \to \tau_2||_{cbn} &= |\tau_1|_{cbn} \to |\tau_2|_{cbn} \\
||\forall t.\sigma||_{cbn} &= \forall t.||\sigma||_{cbn}
\end{aligned}
$$

Call-by-Name Type Transform for contexts

$$|\Gamma|_{cbn}(x) = |\Gamma(x)|_{cbn} \quad \text{for each } x \in \mathrm{dom}(\Gamma)$$

Clarity

$$| - |_{cbn} = | - |$$
$$|| - ||_{cbn} = || - ||$$

Substitution rules

Lemma:

$$|| \, [\tau/t] \, \sigma \, || = [\, ||\tau||/t \,] \, ||\sigma||$$

$$| \, [\tau/t] \, \sigma \, | = [\, ||\tau||/t \,] \, |\sigma|$$

Substitution rules: proof

Lemma:

$$|| \, [\tau/t] \, \sigma \, || = [\, ||\tau||/t \,] \, ||\sigma||$$

$$| \, [\tau/t] \, \sigma \, | = [\, ||\tau||/t \,] \, |\sigma|$$

Proof: by induction on structure of σ.

Meyer-Wand property for DM

Theorem:

1. If $\Gamma \vdash n : \sigma$ holds in DM, then $|\Gamma| \vdash ||n|| : ||\sigma||$ holds in DM
2. If $\Gamma \vdash e : \sigma$ holds in DM, then $|\Gamma| \vdash |e| : |\sigma|$ holds in DM

Meyer-Wand property for DM: proof

Theorem:

1. If $\Gamma \vdash n : \sigma$ holds in DM, then $|\Gamma| \vdash ||n|| : ||\sigma||$ holds in DM
2. If $\Gamma \vdash e : \sigma$ holds in DM, then $|\Gamma| \vdash |e| : |\sigma|$ holds in DM

Proof by induction on the structure of typing derivations

Extension to DM+**cont**

Extend *DM* to $DM + \textbf{cont}$ by defining

$$||\tau \text{ cont}|| = ||\tau|| \to \alpha$$

Then verify

$$||\textbf{callcc}|| : ||\sigma_{\textbf{callcc}}||$$
$$||\textbf{throw}|| : ||\sigma_{\textbf{throw}}||$$

Thank you for your attention