

# Type Theory: Bonus Test about Reading Group

Write your name and student number on each paper that you hand in. This test consists of 3 exercises. The first 10 points are free. The final mark is the number of points divided by 10.

## **Exercise 1 (10+10).**

- (a). Give the definition of a partial combinatory algebra.
- (b). Show that the collection of all  $\lambda$ -terms (not necessarily closed) form a partial combinatory algebra

## **Exercise 2 (10+10+20).** Let $D$ be a partial combinatory algebra.

- (a). Give the definition of a  $D$ -set.
- (b). Give the definition of a modest set.
- (c). Show that whenever  $X$  and  $Y$  are modest sets, then their Cartesian product  $X \times Y$  also is a modest set.

## **Exercise 3 (10+10+20).**

- (a). Give the definition of a polyset structure.
- (b). Let  $X$  and  $Y$  be polysets of some polyset structure  $\mathcal{P}$ . Give the definition of the polyset  $X \rightarrow Y$ .
- (c). Let  $\mathcal{P}$  be a consistent polyset structure. Show that  $X \rightarrow Y \neq \emptyset$  if and only if  $Y = \emptyset$  whenever  $X = \emptyset$ .