An ACL2 Mechanization of an Axiomatic Weak Memory Model

Ben Selfridge

July 13, 2014

Outline

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Multiprocessor Reasoning

Goal: **Analysis of programs** written for **multiple processors** with a shared memory

Multiprocessor Reasoning

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Both approaches have certain advantages and disadvantages

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- Practical MP architectures do not satisfy *sequential consistency* (one reason: write buffers)
- Instead, they satisfy some weaker properties
- **Axiomatic Memory Models** attempt to capture the weaker consistency guarantees of most modern architectures as axioms

Goal of this talk

What this talk is about:

[1] Jade Alglave, Luc Maranget, and Michael Tautschnig. *Herding Cats - Modelling, simulation, testing, and data-mining for weak memory.* To appear in TOPLAS 2014. http://arxiv.org/abs/1308.6810

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- An ACL2 mechanization of this framework
- A new proof of a nice equivalence result for this framework, and a mechanization of this proof
- Ultimate goal: utilize this, or a similar, axiomatic framework to reason about an executable MP model

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How does this translate to concurrent programs?

Concurrent Executions

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Instead, we represent it as a graph, consisting of a collection of **events** with various kinds of directed edges.

Events

Definition

An *event* is a read or a write.

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Components of an event:

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- Type (read or write)

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- Process number

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po is "program order", co is "coherence order", rf is "read-from"

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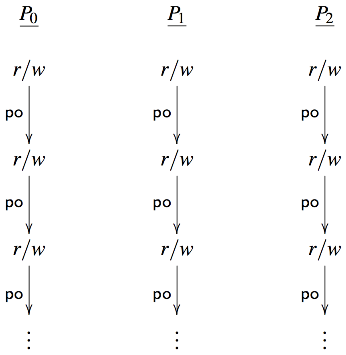
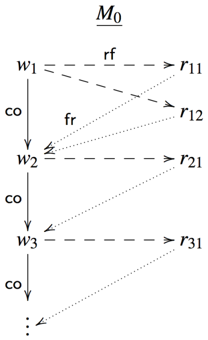
co, rf, and fr are *per-location* dependencies; they relate events which occur at the same memory location only

Concurrent Executions



(a) The per-process view.



(b) The per-location view.

Sequential consistency (SC)

Sequential consistency[2]: "The result of any execution is the same as if the operations of all the processors were executed in some sequential order, and the operations of each individual processor appear in this sequence in the order specified by its program."

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In our framework, we interpret this as the condition

$$\mathsf{acyclic}(\mathsf{po} \cup \mathsf{co} \cup \mathsf{rf} \cup \mathsf{fr})$$

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Modern architectures do not satisfy this constraint.

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SC-Per-Location

Although we don't usually have full sequential consistency, we do have an **analogous notion** that is enforced by most modern architectures:

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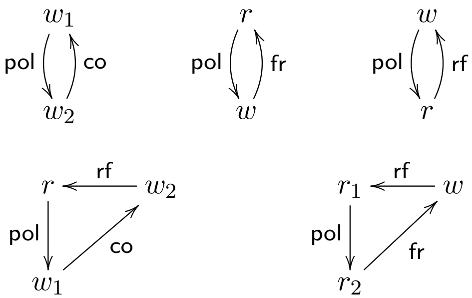
It is **one** of the axioms used in Alglave et. al. [2]

SC-Per-Location

SC-Per-Location is equivalent to prohibiting the following five patterns:



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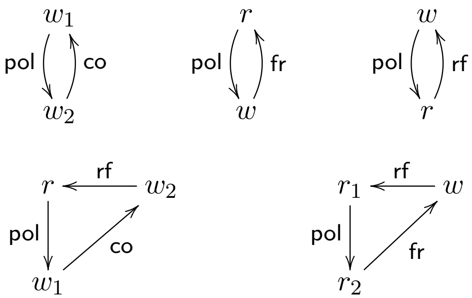

We formalized SC-Per-Location in ACL2 and proved this equivalence.

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- This included a new proof of an equivalence theorem

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An "invalid" graph could be, for instance, one that violates *SC-Per-Location*

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- But we don't want to reason directly about store buffers!
- Unite the model with a higher-level reasoning strategy to handle weak memory more transparently

References

1. Jade Alglave. *A Shared Memory Poetics.* Ph.D. Dissertation. Université Paris 7, 2010.

2. Jade Alglave, Luc Maranget, and Michael Tautschnig. *Herding Cats - Modelling, simulation, testing, and data-mining for weak memory.* To appear in TOPLAS 2014. http://arxiv.org/abs/1308.6810

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- **Operational** semantics have a **closer connection to the actual architecture** being modeled, whereas an axiomatic approach makes a lot of assumptions

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- **Axiomatic** models can be **easier to reason about**; fully modeling an MP architecture can be messy from a theorem-proving perspective