Combining Bialgebraic Semantics and Equations

Jurriaan Rot   Marcello Bonsangue

Leiden University, CWI

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Bialgebras for operational semantics

- mathematical approach to structural operational semantics
- abstracts away from specific syntax and behaviour
- generalize (classical) GSOS, behavioural differential equations

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- abstracts away from specific syntax and behaviour
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Advantages:

- compositional semantics (bisimilarity is a congruence)
- soundness (compatibility) of bisimulation up to context
- ...

Structural operational semantics

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can be modeled by an *abstract GSOS specification*

$$\Sigma(F \times \mathrm{Id}) \Rightarrow FT$$

where

- F is the behaviour functor
- Σ the *signature* (polynomial Set endofunctor)
- TX is $t ::= x \mid \sigma(t_1, \dots, t_n)$

Motivating example (1)

$$\frac{!x \mid x \xrightarrow{a} t}{!x \xrightarrow{a} t}$$

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Problem: how to interpret equations

This research

- These examples do *not* fit in the GSOS format (directly).
- Goal: interpret these in the bialgebraic framework, with the nice well-behavedness properties (bisimilarity a congruence, bisimulation up to context compatible, etc.)

This research

- These examples do *not* fit in the GSOS format (directly).
- Goal: interpret these in the bialgebraic framework, with the nice well-behavedness properties (bisimilarity a congruence, bisimulation up to context compatible, etc.)
- Method: reduce (monotone) specifications *with* equations, to equivalent specifications *without* them.

Interpretation of a GSOS specification

A positive GSOS rule:

$$\frac{\{x_{i_j} \xrightarrow{a_j} y_j\}_{j=1..m}}{\sigma(x_1, \ldots, x_n) \xrightarrow{c} t}$$

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A model ($T\emptyset$ set of closed terms):

$$f : T\emptyset \to \mathcal{P}(A \times T\emptyset)$$

such that $\sigma(t_1, \ldots, t_n) \xrightarrow{a} t'$ iff we can deduce it from one of the rules and the transitions of $t_1, \ldots, t_n$.

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Each GSOS specification has a *unique* model.

Adding assignment rules

$$\frac{\{x_{i_j} \xrightarrow{a_j} y_j\}_{j=1..m}}{\sigma(x_1,\ldots,x_n) \xrightarrow{c} t} \qquad \frac{t \xrightarrow{a} y}{\sigma(x_1,\ldots,x_n) \xrightarrow{a} y}$$

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Example:

$$\frac{\dfrac{a.0 \xrightarrow{a} 0}{!a.0|a.0 \xrightarrow{a} !a.0|0}}{!a.0 \xrightarrow{a} !a.0 \mid 0}$$

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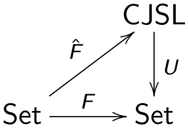
Interpretation: the least model.
For this to exist, we restrict to positive GSOS.

In general

Given an *ordered* behaviour functor F:



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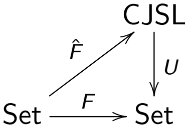

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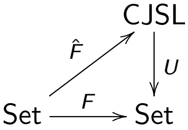

Abstract GSOS specification:

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which is *monotone*.
Assignment rule:

$$\Sigma \Rightarrow T$$

Intepretation, in general

Monotone abstract GSOS specification:

$$\rho : \Sigma(F \times \mathsf{Id}) \Rightarrow FT$$

Assignment rule:

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Model of spec. ρ and set of assignment rules Δ:

$$f : T\emptyset \to FT\emptyset$$

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Intepretation, in general

Monotone abstract GSOS specification:

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Assignment rule:

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Model of spec. ρ and set of assignment rules Δ:

$$f : T\emptyset \to FT\emptyset$$

such that

$$f \circ \iota = F\mu \circ \rho \circ \Sigma\langle f, \mathrm{id}\rangle \vee \bigvee_{d \in \Delta} f \circ \mu \circ d$$

Interpretation: *least* model.

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Idea: get rid of the assignment rules by applying one assignment rule:

$$(1)\ \text{add} \qquad \frac{x \xrightarrow{a} x'}{!x \xrightarrow{a} !x \mid x'} \qquad \text{because} \qquad \frac{\dfrac{x \xrightarrow{a} x'}{!x|x \xrightarrow{a} !x|x'}}{!x \xrightarrow{a} !x \mid x'}$$

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(2) add $\dfrac{x \xrightarrow{a} x'}{!x \xrightarrow{a} !x \mid x' \mid x}$ because $\dfrac{\dfrac{x \xrightarrow{a} x'}{!x \xrightarrow{a} !x|x}}{\dfrac{!x|x \xrightarrow{a} !x|x'|x}{!x \xrightarrow{a} !x|x'|x}}$

etc.

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etc. Use a function φ to do this recursive computation:

$$\varphi(\tau) = \rho \vee \bigvee_{d \in \Delta} (\pi_1 \circ \tau^* \circ d)\,.$$

Main result

Theorem

The interpretation of ρ and Δ coincides with the operational model of the monotone abstract GSOS specification $\mathsf{lfp}\,\varphi$.

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Corollary

Bisimilarity is a congruence on the interpretation of ρ and Δ, and bisimulation up to context is compatible.

Equations

Second part: adding equations to abstract GSOS.

$$\frac{x \xrightarrow{a} x'}{x \mid y \xrightarrow{a} x' \mid y} \quad x \mid y = y \mid x \qquad\qquad !x = !x \mid x$$

Equations

Second part: adding equations to abstract GSOS.

$$\frac{x \xrightarrow{a} x'}{x \mid y \xrightarrow{a} x' \mid y} \quad x \mid y = y \mid x \qquad\qquad !x = !x \mid x$$

Interpreted using *structural congruence*:

$$\frac{t \equiv u \qquad u \xrightarrow{a} u' \qquad u' \equiv v}{t \xrightarrow{a} v}$$

$\equiv$ is the congruence generated by the equations.

Interpretation: transition systems

The *interpretation* is the least model

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2. $\sigma(t_1, \ldots, t_n) \equiv t$ and $t \xrightarrow{a} t'$.

Generalizes to interpreting arbitrary monotone GSOS specification with equations.

Structural congruence: transition systems

Mousavi, Reniers: Congruence for Structural Congruences. FoSSaCS 2005.

- treats structural congruences in the context of structural operational semantics (mainly the tyft/tyxt format)

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- treats structural congruences in the context of structural operational semantics (mainly the tyft/tyxt format)
- provides three different interpretations of structural congruence
- provides a format for equations which guarantees bisimilarity to be a congruence (combined with tyft/tyxt)

Bisimilarity not a congruence

From (Mousavi, Reniers 2005):

$$\frac{}{p \xrightarrow{a} p} \qquad \frac{}{q \xrightarrow{a} p} \qquad p = \sigma(q)$$

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then $p \sim q$, but $\sigma(p) \not\sim \sigma(q)$

A format

(Mousavi, Reniers 2005) cfsc format (w.r.t an SOS specification); equations of the form

$$\sigma(x_1, \ldots, x_n) = \sigma'(y_1, \ldots, y_n)$$

distinct variables, $x_1, \ldots, x_n$ permutation of $y_1, \ldots, y_n$. Or ...

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distinct variables, $x_1, \ldots, x_n$ permutation of $y_1, \ldots, y_n$. Or

$$\sigma(x_1, \ldots, x_n) = t$$

where

- t is a term, variables bound by left-hand side
- $x_1, \ldots, x_n$ distinct
- σ appears nowhere else, in the specification or equations

A theorem

(Mousavi, Reniers 2005): If the equations are in cfsc format and the SOS rules in tyft, then bisimilarity is a congruence.

A generalization

Does it also work with monotone abstract GSOS?

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Encode equations as *assignment rules*. Example:

$$\frac{x \xrightarrow{a} x'}{x \mid y \xrightarrow{a} x' \mid y} \quad x \mid y = y \mid x \qquad\qquad !x = !x \mid x$$

becomes

$$\frac{x \xrightarrow{a} x'}{x \mid y \xrightarrow{a} x' \mid y} \qquad \frac{y \mid x \xrightarrow{a} t}{x \mid y \xrightarrow{a} t} \qquad \frac{!x \mid x \xrightarrow{a} t}{!x \xrightarrow{a} t}$$

Correctness encoding

Suppose E in cfsc format w.r.t. a monotone abstract GSOS specification ρ. Encode E properly as assignment rules Δ.

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Theorem

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If F preserves weak pullbacks, then

- bisimilarity is a congruence
- bisimulation up to context and bisimilarity is compatible

Conclusion

- Treatment of assignment rules and equations in cfsc format, for *monotone* abstract GSOS.
- Abstract specifications with order: notion of proof/derivations

Some future work:

- More on monotone specifications, lookahead in premises
- Monads with equations (Bonsangue, Hansen, Kurz, Rot: CALCO 2013)