Formal languages, grammars, and automata

Assignment 4, Wednesday, Dec. 3 2014

Exercise teachers. Recall the following split-up of students:

teacher	lecture room	email	students
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The delivery boxes are located in the Mercator 1 building on the ground floor (where the Computer Science department ICIS is located).

Handing in your answers. The exercises marked with **points** should be handed in:

1. Delivery box (default): Put your solutions in the appropriate delivery box (see above). Before putting your solutions in the box make sure:
   - your name and student number are written clearly on the document.
2. E-mail (in case your exercise class teacher approves): Send your solutions by e-mail to your exercise class teacher (see above) with subject '*assignment 4*'. This e-mail should only contain a single PDF document as attachment. Make sure:
   - the file is a PDF document that is well readable
   - your name is part of the filename (for example MyName_assignment-4.pdf)
   - your name and student number are in the document (since they will be printed).

Deadline: Monday, December 8, 16:00 sharp!

Goals: After completing these exercises successfully you should be able to use the Pumping Lemma to prove that a language is non-regular and you should be able to use the closure properties for regular languages to show that a language is (non)-regular. The total number of points is 20.

1. Let $\Sigma = \{a, b\}$.
   (a) (**5 points**) Construct a DFA that accepts L where $L = \mathcal{L}(b^*ab^*(ab^*ab^*ab^*)^*)$. (You may write down the DFA directly, without first constructing an NFA.)
   (b) (**5 points**) Derive (just like in the pumping lemma) from your automaton a number k, for which you can prove:
   for every $w \in L$ with $|w| \geq k$, there are words u_1, v, u_2 such that
   - $w = u_1 v u_2$ and
   - $|v| \geq 1$,
   - $|u_1 v| \leq k$
   - $\forall n \in \mathbb{N}(u_1 v^n u_2 \in L)$.

   Prove this.

2. (a) (**5 points**) Prove that the language L_1 is not regular, where

$$L_1 := \{a^n b^p \mid n < p\}$$

   (b) (**5 points**) Prove that the language L_2 is not regular, where

$$L_2 := \{a^n b^p \mid n > p\}$$

   Give a proof using the pumping lemma and (for **2 bonus points**) also try to give a proof without using the pumping lemma (using closure properties of regular languages and languages that we know to be non-regular).

(c) Prove, without using the pumping lemma, that the language L_3 is not regular, where

$$L_3 := \{a^n b^p \mid n \leq p\}.$$

(So only use closure properties of regular languages and languages that we know to be non-regular.)

3. Given L over Σ that is regular, prove that the following language L' is regular:

$$L' := \{w \in \Sigma^* \mid \exists v \in L(w \text{ is a sufffix of } v)\}.$$

NB. w is a *suffix of* v if $v = uw$ for some $u \in \Sigma^*$.

4. Now, $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, \times, (,)\}$ Prove that the language L_4 is not regular, where

$$L_4 := \{e \in \Sigma^* \mid e \text{ is a well-formed arithmetical expression}\}$$

NB. In a *well-formed arithmetical expression* the brackets should "match", so $3 \times (5 + (3 + 0))$ is well-formed and so is $((((4 + 5) \times 7)))$, but $5 + 9) + 3)$ and $(4 \times (3 \times 7)$ are not.

5. (More challenging; This exercise is taken from lecture notes on Languages and Automata by Andy Pitts.)
This exercise shows an example of a language that *can* be pumped, but is *not regular*. Let $\Sigma = \{a, b, c\}$.

(a) Show that the following language can be pumped:

$$L = \{c^m a^n b^n \mid m \geq 1 \text{ and } n \geq 0\} \cup \{a^m b^n \mid m, n \geq 0\}$$

(Show that is satisfies the pumping lemma property with $k = 1$.)

(b) Show that L is non-regular.
[Hint: argue by contradiction. If there is a DFA M accepting L, consider the DFA M' with the same states as M, with Σ just $\{a, b\}$, with transitions all those of M which are labelled by a or b, with start state $\delta(q_o, c)$, (where q_0 is the start state of M), and with the same accepting states. Show that the language accepted by M' is not regular.
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