Formal languages, grammars, and automata
Assignment 7, Wednesday, Jan. 14 2015

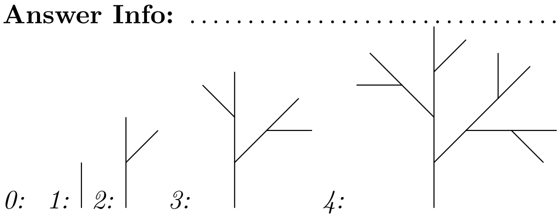
Exercises with answers

1. **(5 points)** Draw 4 iterations of the following Lindenmayer system with start symbol S.

$$S \to F[+S][X]$$
$$X \to F[-X][S]$$

Here + stands for a 45° rotation clockwise, and - stands for a 45° rotation counter-clockwise, F stands for a step forward.

Answer Info: ....



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2. Consider the following context sensitive grammar G_2

S	$\to$	$XSY \mid a \mid b$
Xa	$\to$	aa
Xb	$\to$	bb
Y	$\to$	a

(a) **(5 points)** Describe the language generated by G_2 using set notation.

(b) **(5 points)** Is $\mathcal{L}(G_2)$ context-free? If so, give a CFG.

(c) **(5 points)** Is $\mathcal{L}(G_2)$ deterministic context-free, if so, prove it by giving a deterministic PDA.

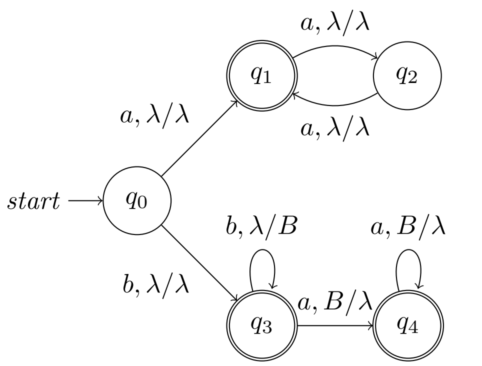
Answer Info: ....

(a) $\mathcal{L}(G_2) = \{a^{2n+1} \mid n \geq 0\} \cup \{b^{n+1}a^n \mid n \geq 0\}$.

(b) $\mathcal{L}(G_2)$ *is context-free, with the following grammar*

S	$\to$	$A \mid B$
A	$\to$	$aAa \mid a$
B	$\to$	$bBa \mid b$

(c) Here is a deterministic PDA



End Answer Info . . . .

3. Consider the languages

   - $L_1 = \{a^n \mid n \text{ is prime}, n \leq 10\}$.
   - $L_2 = \{w \mid w \text{ does not contain } bb\}$.
   - $L_3 = \{wb^n \mid |w|_b = n, n \geq 0\}$

   (a) **(5 points)** One of L_1, L_2, L_3 is not regular, the other two are. Show this by giving regular expressions for these two languages.

   (b) **(5 points)** Show that the other language is not regular using the pumping lemma.

   (c) **(5 points)** Construct a context-free grammar for the language that is not regular.

   (d) **(5 points)** Is $L_2 \cap L_3$ regular? If so, give a regular *grammar* for it.

Answer Info: . . . .

(a) $L_1 = \mathcal{L}(aa + aaa + aaaaa + aaaaaaa)$
$L_2 = \mathcal{L}(a^*(baa^*) * (b + \lambda))$.

(b) *L_3 is not regular.*
Proof by contradiction: suppose that L_3 were regular. Then let k be as in the pumping lemma. Take $w = b^k ab^k \in L_3$. By the pumping lemma $w = uvx$ with $|v| \geq 1$ and $|uv| \leq k$, so uv consists entirely of bs. And also $w' = uvvx = b^{k+|v|}ab^k \in L_3$. But w' is not of the form yb^n, since we would have $n \leq k$ but $|y| > k$. Therefore $w' \notin L_3$. This is a contradiction, so L_3 must not be regular.

(c) *L_3 is generated by*

$$\begin{array}{lcl} S & \rightarrow & AbSb \mid A \\ A & \rightarrow & Aa \mid \lambda \end{array}$$

Here A generates 0 or more as. S is like the grammar for $a^n b^n$, but with as allowed in the left part, and bs instead of as.

(d) *Yes, in $L_2 \cap L_3$, n must be 0 or 1, otherwise the string contains two consecutive bs. So $L_2 \cap L_3 = \mathcal{L}(a^* + a^*baa^*b)$. As a regular grammar we can write*

$$\begin{array}{lcl} S & \rightarrow & aS \mid bA \mid \lambda \\ A & \rightarrow & aB \\ B & \rightarrow & aB \mid bC \\ C & \rightarrow & \lambda \end{array}$$

End Answer Info . . . .