Talen en Automaten, NWI-IPC002

Retake exam, Wed 13th Apr, 2016
18:00 – 21:00

This test consists of **six** exercises over **3 pages**. Explain your approach. You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A and $a \in A$ by $|w|_a$ the number of a's in the word $w \in A^*$, as it was introduced in the lecture.

1. Consider the regular expression $e_1 = (\mathtt{a}+\mathtt{b})^*(\mathtt{b}+\mathtt{c})^*(\mathtt{c}+\mathtt{a})^*$ over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

   a) Give the shortest word over Σ that is *not* in $\mathcal{L}(e_1)$ and explain your answer. **(5pt)**

   b) Construct a nondeterministic automaton with λ-transitions, i.e. an NFA$_\lambda$, that accepts $\mathcal{L}(e_1)$. Explain your answer. **(7pt)**

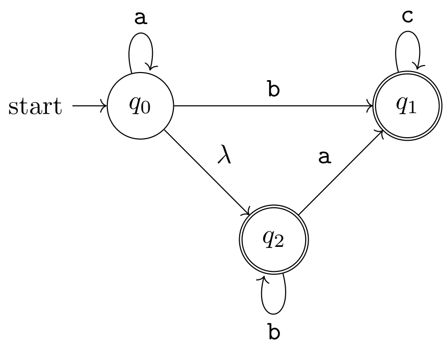
2. Consider the language $L_2 = \{w \mid w \text{ contains the substring } \mathtt{bbb}\}$ over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Here, u is a substring of w, if $w = xuy$ for some words x, y.

   a) Give a regular expression for L_2 and explain your answer. **(6pt)**

   b) Give a deterministic finite automaton (DFA) that accepts L_2. **(6pt)**

   c) Give a regular grammar that generates L_2. **(3pt)**

3. Let M_3 be the NFA$_\lambda$ over the alphabet $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and the following state diagram:



   a) Compute a regular expression e_3 such that $\mathcal{L}(e_3) = \mathcal{L}(M_3)$ using the elimination-of-states algorithm. **(5pt)**

   b) Construct a DFA (deterministic finite automaton) M_3' using the subset construction that accepts the same language as M_3. Label the states in M_3' so that it is clear which states in M_3 they correspond to. **(8pt)**

4. Let G_4 be the following context free grammar over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

$$\begin{aligned} S &\rightarrow \mathtt{a}S \mid ASA \mid SC \mid \lambda \\ A &\rightarrow \mathtt{ab}A \mid \lambda \\ C &\rightarrow \mathtt{c}C \mid \lambda \end{aligned}$$

a) Give a leftmost derivation in G_4 of abacab. **(3pt)**

b) Is this grammar ambiguous? Explain your answer. **(3pt)**

c) Is $\{w \in \Sigma^* \,:\, |w|_{\mathtt{b}} \leq |w|_{\mathtt{a}}\} \subseteq \mathcal{L}(G_4)$? Explain your answer. **(4pt)**

d) For which $m, n, k \geq 0$ do we have $\mathtt{a}^m\mathtt{b}^n\mathtt{c}^k \in \mathcal{L}(G_4)$? Explain your answer. **(4pt)**

5. Let L_5 be the language

$$L_5 = \{w\mathtt{a}v \mid w, v \in \{\mathtt{a}, \mathtt{b}\}^* \text{ and } |w| \leq |v|\}.$$

a) Give a context-free grammar G_5 that generates L_5. **(7pt)**

b) Provide derivations in G_5 of the words: abbb, bbaaaa, baaaab. **(3pt)**

c) Give a pushdown automaton M_5 that accepts L_5. **(8pt)**

d) Provide accepting computations of M_5 of the words: aab, aba, babaaa. **(3pt)**

e) Prove that L_5 is not regular. **(10pt)**

6. Let $M_6 = (Q, \Sigma, \Gamma, \delta, q_0, F)$ be the PDA with

$Q = \{q_0, q_1, q_2\}$	$\delta(q_0, \mathtt{a}, \lambda) = \{(q_0, A)\}$
$\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$	$\delta(q_0, \mathtt{a}, A) = \{(q_0, \lambda)\}$
$\Gamma = \{A, B, C\}$	$\delta(q_0, \lambda, \lambda) = \{(q_1, \lambda)\}$
$F = \{q_0\}$	$\delta(q_1, \mathtt{b}, \lambda) = \{(q_1, B)\}$
	$\delta(q_1, \mathtt{c}, B) = \{(q_2, C)\}$
	$\delta(q_2, \lambda, C) = \{(q_0, \lambda)\}$

a) Draw the state diagram of M_6. **(6pt)**

b) Check which of the following words are in $\mathcal{L}(M_6)$ and explain your answer by giving a computation or showing that there is none: aaaa, abc, abbcca. **(3pt)**

c) Describe $\mathcal{L}(M_6)$ using set notation. That is, write **(6pt)**

$$\mathcal{L}(M_6) = \{\ldots\},$$

with ... a precise mathematical description of the words accepted by M_6.

More exercises (from re-exam 2017-2018)

1. For each of the following claims, state whether it is true or false. Support your claim with either a counterexample or a (short) argument/proof.

   (a) If a language L contains finitely many words, it is regular.

   (b) If a language is context-free, it is not regular.

   (c) The language $L = \mathcal{L}\left(b(a+b)^*\right) \cup \{a^n b^m \mid n \neq m\}$ is regular.

2. Let $A = \{a, b\}$ be our finite alphabet and define $f : A^* \to A^*$ inductively as follows:

$$f(\lambda) = \lambda \qquad f(av) = f(v)aba \qquad f(bv) = f(v)bb$$

   Prove that $f(wa) = abaf(w)$ for all $w \in A^*$, by induction on w.