Talen en Automaten
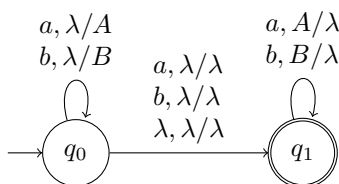## Additional assignments for exercise class on Fri 14$^{\text{th}}$ Dec, 2018

a) Recall the language L of palindromes:

$$L = \{w \in \{a, b\}^* \mid w = w^R\}.$$

Give a PDA that accepts L, and show that aba is accepted but ab is not.
Solution: ....

Let M be the PDA with stack alphabet $\{A, B\}$ be given as follows.

Then M accepts aba by

$$(q_0, aba, \lambda) \Rightarrow (q_0, ba, A) \Rightarrow (q_1, a, A) \Rightarrow (q_1, \lambda, \lambda).$$

Further, ab is rejected because we have the following computations for it:

$$(q_0, ab, \lambda) \Rightarrow \{(q_0, b, A), (q_1, b, \lambda), (q_1, ab, \lambda)\}$$
$$\Rightarrow \{(q_0, \lambda, BA), (q_1, \lambda, A), (q_1, b, A)\}$$

and every triple here is stuck. Thus there is no computation $(q_0, ab, \lambda) \Rightarrow^* (q_1, \lambda, \lambda)$, hence ab is not accepted. □

b) Use the algorithm from the lecture to construct a CFG for the language L of palindromes, using the PDA in your answer to the previous question.

Solution: ....

Following the algorithm, we get the following grammar:

$$
\begin{aligned}
S &\rightarrow (q_0, q_1) \\
(q_0, q_0) &\rightarrow \lambda \mid (q_1, q_0) \mid a(q_1, q_0) \mid b(q_1, q_0) \mid a(q_0, q_1)a(q_1, q_0) \mid b(q_0, q_1)b(q_1, q_0) \\
(q_0, q_1) &\rightarrow (q_1, q_1) \mid a(q_1, q_1) \mid b(q_1, q_1) \mid a(q_0, q_1)a(q_1, q_1) \mid b(q_0, q_1)b(q_1, q_1) \\
(q_1, q_1) &\rightarrow \lambda
\end{aligned}
$$

□

c) Use the construction from the lecture to give a PDA accepting the language generated by the following grammar:
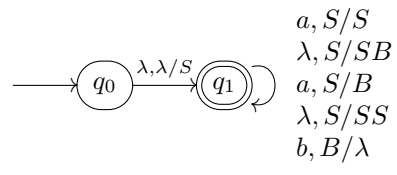
$$S \quad \rightarrow \quad a\,S \mid S\,b \mid a\,b \mid S\,S$$

Solution: ....

First, put the grammar in the right form:

$$
\begin{aligned}
S &\rightarrow a\,S \mid S\,B \mid a\,B \mid S\,S \\
B &\rightarrow b
\end{aligned}
$$

The PDA is:

$$a, S/S$$
$$\lambda, S/SB$$
$$a, S/B$$
$$\lambda, S/SS$$
$$b, B/\lambda$$

$$\xrightarrow{\quad} q_0 \xrightarrow{\lambda, \lambda/S} q_1 \circlearrowleft$$

□