Talen en Automaten

Retake Exam, Wed 1st Mar, 2017
18:00 – 21:00

This test consists of **5** problems over **2 pages**. Explain your approach, and **write your answers to the exercises on a separate folio (double pages) as indicated.** You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A, $w \in A^*$ and $a \in A$ by $|w|_a$ the number of a's in w, as it was introduced in the lecture.

Write your answers to Problems 1 and 2 on a separate folio (double page)

Problem 1.

Let $A = \{a, b, c\}$ be our finite alphabet and define $f : A^* \to A^*$ inductively as follows

$$f(\lambda) = \lambda \qquad f(a\,v) = a\,b\,f(v) \qquad f(b\,v) = b\,c\,f(v) \qquad f(c\,v) = c\,a\,f(v)$$

a) Give a word w for which $f(w) = abcabc$. **(3pt)**

b) Prove by induction the following property (for all $w \in A^*$) **(8pt)**

$$|f(w)|_a + |f(w)|_b = 2|w|_a + |w|_b + |w|_c.$$

Problem 2.

Consider the following regular grammar for well-formed natural number expressions. The alphabet is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and N is the start symbol. (Note that we avoid leading zeroes, so 34 and 304 are well-formed, but 034 is not.)

$$\begin{array}{lcl} N & \to & 0X \mid 1A \mid 2A \mid ... \mid 9A \\ A & \to & 0A \mid 1A \mid 2A \mid ... \mid 9A \mid \lambda \\ X & \to & \lambda \end{array}$$

a) Give a context free grammar for the language L of *well-formed arithmetic expressions*, that is expression involving natural numbers (see above), the binary operation $\oplus$ and brackets. **(10pt)**
So the alphabet is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, \oplus, (,)\}$ and you need to describe the expressions where the "brackets match" and $\oplus$ is a binary operation. For example, $33 \oplus 33$, $((33) \oplus 34)$, $33 \oplus 34 \oplus 34$ and $((33))$ are well-formed, but $33 \oplus 4)$, $33) \oplus 4$, $33(\oplus 44)$ and $\oplus \oplus 33$ are not.

b) Prove that L is not regular. **(10pt)**

c) Give a regular grammar for the language L' of well-formed arithmetic expressions over natural numbers (see above) without brackets. **(8pt)**
So the alphabet is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, \oplus\}$. For example, $33 \oplus 33$ and $33 \oplus 34 \oplus 34$ are well-formed, but $33\oplus$ and $\oplus \oplus 33$ are not.

d) Give a regular expression for the language L' (of the previous item). **(8pt)**

Write your answers to Problems 3, 4, 5, 6 on a separate folio (double page)

Problem 3.

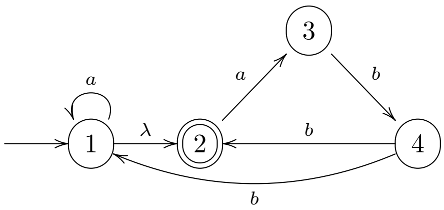
Consider the language L over the alphabet $A = \{0, 1\}$, defined by **(8pt)**

$$L = \{w \mid w \text{ starts with 00 and does } \textit{not} \text{ end with 00}\}$$

Give a DFA that accepts L. Explain your answer.

Problem 4.

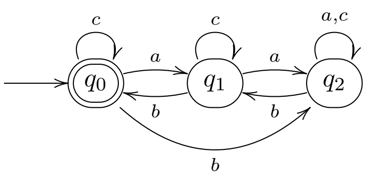
Let $A = \{a, b\}$ and consider the following NFA-λ M over A. **(10pt)**



Use the powerset construction to give a DFA D with $\mathcal{L}(D) = \mathcal{L}(M)$. Indicate clearly from which states in M a state in D originates.

Problem 5.

Let $A = \{a, b, c\}$ and the DFA M over A given by **(10pt)**



Use the procedure from the lecture to construct a regular expression e such that $\mathcal{L}(M) = \mathcal{L}(e)$. Show each intermediate step.

Problem 6.

Let M be the PDA with

$Q = \{q_0, q_1, q_2\}$	$\delta(q_0, a, \lambda) = \{\langle q_0, A\rangle\}$	$\delta(q_0, \lambda, \lambda) = \{\langle q_1, \lambda\rangle\}$
$\Sigma = \{a, b\}$	$\delta(q_1, a, A) = \{\langle q_1, \lambda\rangle\}$	$\delta(q_1, b, A) = \{\langle q_1, \lambda\rangle\}$
$\Gamma = \{A\}$	$\delta(q_1, \lambda, A) = \{\langle q_2, \lambda\rangle\}$	$\delta(q_2, \lambda, A) = \{\langle q_2, \lambda\rangle\}$
$F = \{q_2\}$		

a) Draw a state diagram for M. **(4pt)**

b) Show that aab is accepted by M, but that $aabb$ is not. **(5pt)**

c) Give a precise description of $\mathcal{L}(M)$ using set notation. **(8pt)**

d) Construct a CFG G such that $\mathcal{L}(M) = \mathcal{L}(G)$. Explain your answer. **(8pt)**