Talen en Automaten

Test 2, Wed 18th Jan, 2017
8h30 – 11h30

This test consists of **5** problems over **2 pages**. Explain your approach, and **write your answers to the exercises on a separate folio (double pages) as indicated.**. You can score a maximum of 100 points, and each question indicates how many points it is worth. The test is closed book. You are NOT allowed to use a calculator, a computer or a mobile phone. You may answer in Dutch or in English. Please write clearly, and do not forget to put on each page: your name and your student number.

Notation Throughout the test, we denote for any alphabet A, $w \in A^*$ and $a \in A$ by $|w|_a$ the number of a's in w, as it was introduced in the lecture. Moreover, recall that v is a *subword* of w if $w = xvy$ for some words x, y.

Write your answers to Problems 1 and 2 on a separate folio (double page)

Problem 1.

Consider the following languages over the alphabet $A := \{a, b, c\}$.

- $L_1 = \{wcvcz \mid w, v, z \in \{a, b\}^* \text{ and }, |w|_a = |v|_a = |z|_a\}$.
- $L_2 = \{w \mid w \text{ does not contain } bb \text{ as subword}\}$.
- $L_3 = \{wb^n \mid |w|_b = n, n \geq 0\}$

One of L_1, L_2, L_3 is regular, one is context-free but not regular and one is not context free.

a) Which of the languages is regular? Show this by giving a regular grammar for this language. **(8pt)**

b) Which of the languages is context free but not regular? Give a context free grammar for this language. **(8pt)**

c) Is $L_2 \cap L_3$ regular? If so, give a regular grammar for it. Otherwise argue that it is not regular. (You don't have to give a full proof.) **(8pt)**

Problem 2.

Consider the following context-free grammar G over $\{a, b, c\}$.

$$\begin{aligned} S &\to a\,S\,b \mid C\,X \mid \lambda \\ C &\to c\,C \mid \lambda \\ X &\to S\,c \end{aligned}$$

a) Indicate for the following words if they are generated by G: ab, $aabba$, $abab$. Explain your answer. (So give a derivation in case the word is in $\mathcal{L}(G)$ and otherwise give an argument why it is not.) **(6pt)**

b) Use the procedure from the lecture to construct a PDA (push-down automaton) that accepts the language generated by G. **(8pt)**

c) Can all words of the shape $(ac)^n ab(cb)^n$ (with $n \geq 0$) be produced by G_1? Prove your answer. **(7pt)**

Write your answers to Problems 3 and 4 on a separate folio (double page)

Problem 3.

Consider the following language over the alphabet $A = \{a, b\}$.

$$L = \{(ab)^n w \mid w \text{ contains } n \text{ copies of } ab \text{ as subword, for some } n \geq 0\}$$

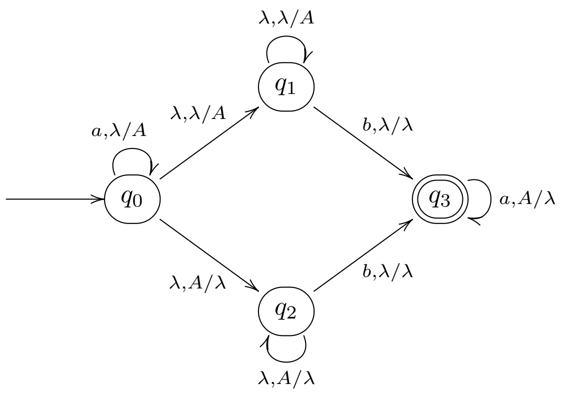
a) Give a PDA that accepts L. **(10pt)**

b) Show that $abaabb$ and $abbab$ are accepted by your automaton, by giving the accepting computations. **(4pt)**

c) Show that $ababab$ is not accepted by your automaton. **(4pt)**

Problem 4.

We define the PDA M, with input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{A\}$, as follows.



a) Show that $aaba$ and $abaa$ are accepted by M. **(4pt)**

b) Show that $aabaa$ is not accepted by M. **(4pt)**

c) Is M deterministic? Explain your answer. **(4pt)**

d) Give a precise description of $\mathcal{L}(M)$ using set notation. **(10pt)**

Write your answers to Problem 5 on a separate folio (double page)

Problem 5.

Let M be the PDA with

$Q = \{q_0, q_1\}$	$\delta(q_0, a, \lambda) = \{\langle q_0, A\rangle\}$
$\Sigma = \{a, b\}$	$\delta(q_0, b, \lambda) = \{\langle q_1, \lambda\rangle\}$
$\Gamma = \{A\}$	$\delta(q_1, a, A) = \{\langle q_1, \lambda\rangle\}$
$F = \{q_1\}$	$\delta(q_1, b, \lambda) = \{\langle q_1, \lambda\rangle\}$

a) Draw a state diagram for M. **(5pt)**

b) Construct a CFG G such that $\mathcal{L}(M) = \mathcal{L}(G)$. Explain your answer. **(10pt)**