1 Counting Letters

a) Let A be a non-empty, finite alphabet and $a \in A$ a letter.

Define by structural induction a map **(1pt)**

$$|\,.\,|_a \colon A^* \to \mathbb{N}$$

that counts the number of occurrences of the letter a in a word.

b) Show, by induction, that for any two words $w, u \in A^*$ **(2pt)**

$$|wu|_a = |w|_a + |u|_a.$$

2 Regular expression

Let L be the language given by

$$\left\{ w \in \{a, b\}^* \,\middle|\, \text{every } b \text{ in } w \text{ is directly followed by an } a \right\}$$

Give a regular expression for the language L and explain your answer. **(1pt)**