

Talen en Automaten

Additional assignments for exercise class on Fri 24th Nov, 2017

a) Let L be the following language over $A = \{a, b\}$.

$$L = \{w \in A^* \mid |w|_a \text{ is even and } w \text{ does not contain } bb \text{ as a subword}\}$$

i) Use the constructions for product and complement automata, given in the lecture, to construct an automaton M with $\mathcal{L}(M) = L$.

Solution:

We first note that

$$L = L_1 \cap \overline{L_2}$$

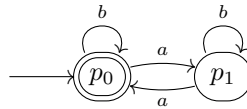
where

$$L_1 = \{w \in \{a, b\}^* \mid |w|_a \text{ is even}\}$$

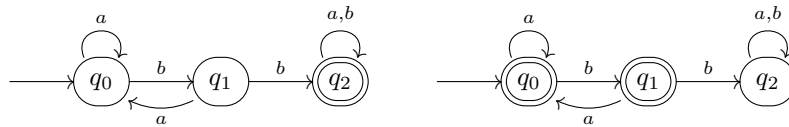
and

$$L_2 = \{w \in \{a, b\}^* \mid w \text{ contains } bb \text{ as a subword}\}$$

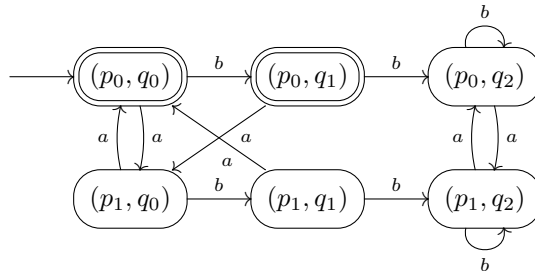
We give an automaton for L_1 :



and an automaton for L_2 (on the left) and its complement $\overline{L_2}$ (on the right):



Now we construct the product of the automata for L_1 and $\overline{L_2}$



□

ii) Give the accepting computation in M for $abab$, and show that M does not accept abb .

Solution:

For $abab$, we start as above;

$$(p_0, q_0), abab \vdash (p_1, q_0), bab \vdash (p_1, q_1), ab \vdash (p_0, q_0), b \vdash (p_1, q_0), \lambda$$

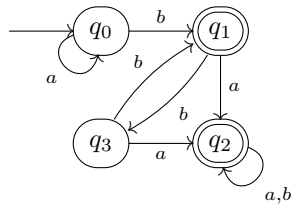
and (p_1, q_0) is accepting. For abb :

$$(p_0, q_0), abb \vdash (p_1, q_0), bb \vdash (p_1, q_1), b \vdash (p_1, q_2), \lambda$$

and (p_1, q_2) is not accepting.

□

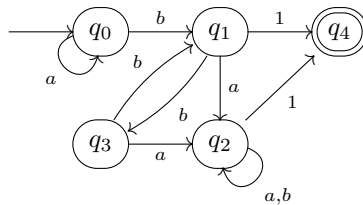
b) Consider the following automaton, which we call M .



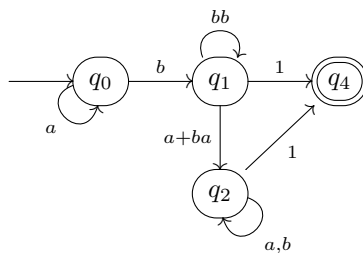
Use the state elimination method to construct a regular expression e such that $\mathcal{L}(e) = \mathcal{L}(M)$.

Solution:

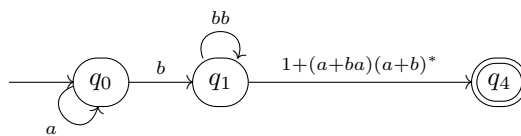
We first add a single accepting state.



We remove q_3 :

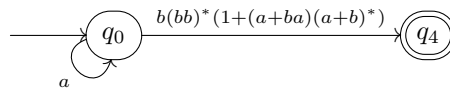


We remove q_2 :



We remove q_1 :

We remove q_2 :



We conclude with the regular expression

$$a^*b(bb)^*(1 + (a + ba)(a + b)^*)$$

□