Talen en Automaten

Additional assignments for exercise class on Fri 8th Dec, 2017

1 Non-regular languages via closure properties

Show that $L = \{wv \in \{a,b\}^* \mid |w| = |v| \text{ and } w \neq v\}$ is not regular.

Solution: ....

Suppose L were regular, then by the closure properties also

$$K = L \cup \{w \in \{a,b\}^* \mid |w| \text{ is odd }\}$$

would be regular. Hence, the complement $\overline{K}$ would be regular. But it is easy to check that

$$\overline{K} = \{ww \mid w \in \{a,b\}^*\}$$

This language has been shown to be non-regular in the lecture. Thus we arrive at a contradiction and L cannot be regular. □

2 Pumping lemma

Show that the language $L = \{ucv \mid u, v \in \{a,b\}^*, u \text{ appears as subword in v}\}$ over the alphabet $\{a,b,c\}$ is not regular.

Solution: ....

Suppose L were regular, then the pumping lemma applies to it; let k be the constant from the pumping lemma. We let $w = a^k c a^k$. Clearly $w \in L$, since a^k is a subword of itself. Now let $w = xyz$ be a decomposition from the pumping lemma; then $|xy| \leq k$ and $|y| \geq 1$. Then $x = a^i$ and $y = a^j$ for some i, j. By the pumping lemma, we have $xy^2z \in L$. But $xy^2z = a^i a^{2j} a^{k-(i+j)} c a^k = a^{k+j} c a^k$. And since $j + k > k$ (remember that $j > 0$), we have $xy^2z \notin L$, contradiction. □