

Talen en Automaten

Additional assignments for exercise class on Fri 19th Jan, 2018

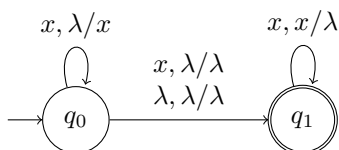
a) Recall the language L of palindromes:

$$L = \{w \in A^* \mid w = w^R\}.$$

Give a PDA that accepts L , and show that aba is accepted but ab is not.

Solution:

Let M be the PDA with stack alphabet A be given as follows, where $x \in A$.



Then M accepts aba by

$$(q_0, aba, \lambda) \Rightarrow (q_0, ba, a) \Rightarrow (q_1, a, a) \Rightarrow (q_1, \lambda, \lambda).$$

Further, ab is rejected because we have the following computations for it:

$$\begin{aligned} (q_0, ab, \lambda) &\Rightarrow \{(q_0, b, a), (q_1, b, \lambda), (q_1, ab, \lambda)\} \\ &\Rightarrow \{(q_0, \lambda, ba), (q_1, \lambda, a), (q_1, b, a)\} \end{aligned}$$

and $(q_1, b, \lambda) \not\Rightarrow^*$. Thus there is no computation $(q_0, ab, \lambda) \Rightarrow^*(q_1, \lambda, \lambda)$, hence ab is not accepted. □

b) Use the algorithm from the lecture to construct a CFG for the language L of palindromes, using the PDA in your answer to the previous question.

Solution:

Following the algorithm, we get the following grammar:

$$\begin{aligned} S &\rightarrow (q_0, q_1) \\ (q_0, q_0) &\rightarrow \lambda \mid (q_1, q_0) \mid a(q_1, q_0) \mid b(q_1, q_0) \mid a(q_0, q_1)a(q_1, q_0) \mid b(q_0, q_1)b(q_1, q_0) \\ (q_0, q_1) &\rightarrow (q_1, q_1) \mid a(q_1, q_1) \mid b(q_1, q_1) \mid a(q_0, q_1)a(q_1, q_1) \mid b(q_0, q_1)b(q_1, q_1) \\ (q_1, q_1) &\rightarrow \lambda \end{aligned}$$

□

c) Use the construction from the lecture to give a PDA accepting the language generated by the following grammar:

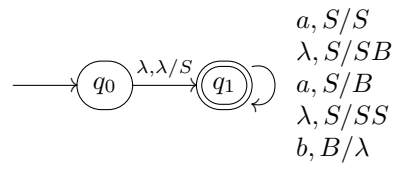
$$S \rightarrow aS \mid Sb \mid ab \mid SS$$

Solution:

First, put the grammar in the right form:

$$\begin{aligned} S &\rightarrow aS \mid SB \mid aB \mid SS \\ B &\rightarrow b \end{aligned}$$

The PDA is:



□