

Talen en Automaten

Assignment 5, Tuesday 19th December, 2017

Exercise teachers. The student groups are supervised by the following teachers:

Teacher	E-Mail	Room	Time
Michiel de Bondt	M.deBondt@math.ru.nl	HG00.310	8:45 – 10:30
Demian Janssen	wd.janssen@student.ru.nl	HG01.058	8:45 – 10:30
Leon Gondelman	lgondelmann@gmail.com	HG00.065	8:45 – 10:30
Tom van Bussel	tom.van.bussel@student.ru.nl	HG02.028	8:45 – 10:30
David Venhoek	david@venhoek.nl	HG03.632	8:45 – 10:30
Alexis Linard	A.linard@cs.ru.nl	HG00.058	8:45 – 10:30
Bas Steeg	bas.steeg@student.ru.nl	HG00.062	10:45 – 12:30
Ties Robroek	ties.robroek@student.ru.nl	HG00.062	10:45 – 12:30
Jan Martens	j.martens@student.ru.nl	HG00.308	15:45 – 17:30
Nienke Wessel	N.Wessel@student.ru.nl	HG00.310	15:45 – 17:30

Postboxes are located in the Mercator building on the ground floor. There are 7 boxes labelled with *Talen en Automaten* and the corresponding group teacher's name. There is 1 box, the *Uitleverbak*, for work that hasn't been picked up at the exercise hours.

Handing in your answers: There are two options:

1. E-mail: Send your solutions by e-mail to your exercise class teacher (see above) with subject “**T&A: assignment 5**”. This e-mail should only contain a single PDF document as attachment (unless explicitly stated otherwise). Before sending an e-mail make sure:
 - the file is a PDF document
 - your name is part of the filename (for example MyName_assignment-5.pdf)
 - your name and student number are included in the document (they will be printed).
2. Post box: Put your solutions in the appropriate post box (see above). Before putting your solutions in the post box make sure:
 - your name, student number, and IC, KI or Wiskunde are written clearly on the document.

Deadline: Tuesday 9th January, 2018, 8:45 (in Nijmegen!)

Goals: After completing these exercises successfully you should be able to read context-free grammars, as well as write down grammars for context-free languages and regular languages. The total number of points is 10.

There are 3 mandatory exercises, worth **10 points** in total. There are 3 more, extra hard, exercises. Be aware that these exercises are just for fun, you cannot earn any points with them.

1 Ambiguous grammars

Let $\Sigma = \{a, b\}$. Consider the following context-free grammar

$$G_1 = \boxed{S \rightarrow aS \mid Sb \mid ab \mid SS}$$

- a) Show that the grammar is ambiguous by giving two left-most derivations of $aabb$. **(1pt)**

b) Give a regular expression for $L_1 := \mathcal{L}(G_1)$. (1pt)

c) Give a non-ambiguous regular grammar for L_1 . (1pt)

2 Constructing context-free grammars

For each of the following languages construct a context-free grammar that generates the language, and **explain why your answer is correct**.

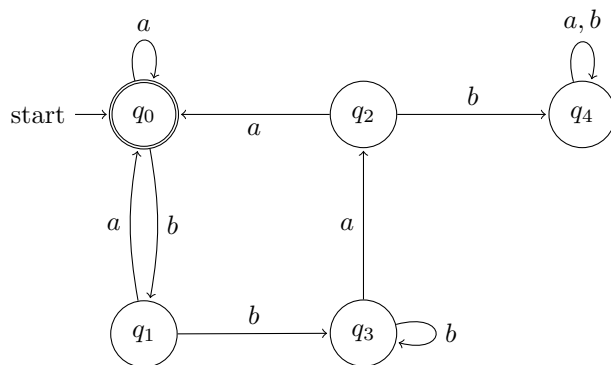
(2 points) $L_2 = \{a^n b^{n+m} a^m \mid n, m \geq 0\}$

(2 points) $L_3 = \{w \in \{a, b, r\}^* \mid \text{every } a \text{ (in } w) \text{ is followed directly by } b, \text{ and } w \text{ ends with } brrr\}$

(2 points) $L_4 = \mathcal{L}((aa^*b)^*a(aa+bb)^*)$

3 From DFA to regular grammar

Consider the following DFA M :



Construct a context-free grammar that generates $\mathcal{L}(M)$. (1pt)

Fun Exercises

1. Construct a context-free grammar that generates the following language:

$$L_5 = \{w \in \{a, b\}^* \mid |w| = 2k + 1 \text{ and } w_1 = w_{k+1}\},$$

where w_i denotes the i -th symbol in a word w . That is, L_5 consists of all words of odd length that have the same symbol in the first and middle positions.

2. $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \times, +, (,)\}$. Construct a context-free grammar that generates the language

$$L_6 = \{w \in \Sigma^* \mid w \text{ is a well-formed arithmetical expressions}\}$$

NB. $2 + 3 + 4 \times 5$ and $((2 + 3) + 4) \times 5$ and $((2 + 3)) + 4 \times 5$ are well-formed. $2 + (3 + 4 \times 5$ and $(2 + 3) + 4) \times 5$ and $) ($ are not.

3. Suppose that L and L' are context-free languages. Show that both L^* and LL' are context-free languages.