

Description Logics and Frames

- Considerable interest in this topics
 - seen as starting point for representing **huge quantities** of knowledge
 - often used for representing terminologies in particular domains
 - any increase of level of automation will give rise to an increased significance in domain description



- **Semantic web:**
OWL DL (Web Ontology Language/Description Logic): formal semantics (model theory) and support for reasoning

<http://www.w3.org/TR/owl2-overview/>

Basic ideas are old ...



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Stoeh^{olm} Berolⁱⁿ Monsp^{is} et Paris^{ensis} Soc^{ietatis}
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J. M. Aronsson del. J. G. W. Lundström sculp.

J. G. W.

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ARCHIATR. REG. MED. ET BOT. PROFESS. UPSAL.
SYSTEMA
NATURÆ
SISTENS
REGNA TRIA NATURÆ,
IN
CLASSES ET ORDINES
GENERA ET SPECIES
REDACTA
TABULISQUE ÆNEIS ILLUSTRATA.



CUM PRIVILEGIO S. R. M. POLONICÆ AC ELECTORIS SAXON.
Secundum sextam STOCKHOLMIENSEM emendatam & auctam
editionem.

LIPSIÆ,
IMPENSIS GODOFR. KIESEWETTERI.

1748.

Ontologies

- **Ontology**: science which investigates and explains the nature and essential properties and relations of all beings (Aristotle: *Metaphysica*)
- In AI, artifact that
 - makes use of a vocabulary,
 - with a set of rules about syntax and meaning,
 - purpose: **computer-interpretable** description of a domain
- Typical building blocks:
 - **names** (concepts)
 - **relations** and **constraints**
- Popular applications: in biomedical research (e.g. <http://bioontology.org/>)

Linnaeus

Ordo I.

PRIMATES.

Dentes primores superiores IV paralleli.
Mammæ pectorales, binæ.

I. HOMO nosce Te ipsum.

1. H. diurnus. (*) *vagans cultura, loco.*
 - a. H. rufus, cholericus, rectus. Americana.
 - β. H. albus, sanguineus torolus. Europæus.
 - γ. H. luridus, melancholicus rigidus. Asiaticus.
 - δ. H. niger, phlegmaticus, laxus. Afer.
 - ε. H. monstruosus solo (a), vel arte (b. c.)
 - a. *Alpini parvi, agiles, timidi: Patagonici magni, segnes.*
 - b. *Monorchides ut minus fertiles: Hottentotti.*
Juncæ puellæ abdomine attenuato: Europæ.
 - c. *Macrocephali capite conico. Chineses.*
Plagiocephali capite antice compresso. Canadenses.
2. Homo nocturnus. Ourang Outang *Bont. jav. 84. t. 84.*

Genus Trogloditæ seu Ourang Outang ab Homine vero diffi-
ciam, adhibita quamvis omni attentione, obtinere non potui, nisi as-
merem notam lubricam, in aliis generibus non constantem. Nec Den-
tes lanarii minime a reliquis remoti; nec Nymphae callæ, quibus
carent Simiæ, hinc ad Simias reducere admittebant. Inquirant as-
toptæ in vivo, qua ratione, modo notæ aliqua existant, ab Hominis
genere separari queat, nam inter Simias versantem oportet esse Si-
miam. Apolloder.

Cycorp

The screenshot shows a Mozilla Firefox browser window displaying the Cycorp website. The address bar shows the URL <http://www.cyc.com/cyc/company/company>. The browser's menu bar includes File, Edit, View, History, Bookmarks, Tools, and Help. The website's navigation menu includes News, Downloads, Software, Hardware, Developers, Help, Search, and Shop. The main content area features a breadcrumb trail: home > company > about Cycorp. A large, abstract, purple-toned image serves as a background for the 'about Cycorp' section. The Cycorp logo, a purple knot-like symbol above the text 'cycorp', is positioned on the right. Below the logo is a vertical list of links: about Cycorp, media coverage, staff, and employment. A circular image of a dense, textured, black and white pattern is located to the left of a quote. The quote reads: "Once you have a truly massive amount of information integrated as knowledge, then the human-software system will be superhuman, in the same sense that mankind with writing is superhuman compared to mankind before writing." ~ Doug Lenat, June 21, 2001. Below the quote is a paragraph of text describing Cycorp as a leading provider of semantic technologies. Another paragraph describes Cycorp as a premier knowledge-based technologies research and development company. A final paragraph details the history of the Cyc project, founded in 1984 by Dr. Douglas Lenat. The browser's status bar at the bottom shows the word 'Done'.

Cycorp, Inc. - Mozilla Firefox

File Edit View History Bookmarks Tools Help

[http://www.cyc.com/cyc/company/company](#) G Cyc


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Cycorp, Inc.


home > company > about Cycorp

about Cycorp

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- about Cycorp
- media coverage
- staff
- employment



"Once you have a truly massive amount of information integrated as knowledge, then the human-software system will be superhuman, in the same sense that mankind with writing is superhuman compared to mankind before writing." ~ Doug Lenat, June 21, 2001

Cycorp is a leading provider of semantic technologies that bring a new level of intelligence and common sense reasoning to a wide variety of software applications. The Cyc® software combines an unparalleled common sense ontology and knowledge base with a powerful reasoning engine and natural language interfaces to enable the development of novel knowledge-intensive applications.

As a premier knowledge-based technologies research and development company, Cycorp leverages its cutting edge innovations in knowledge representation, machine reasoning, natural language processing, semantic data integration, and information management and search to offer an array of semantic middleware, knowledge-based application development capabilities, and turn-key solutions.

The Cyc project, a long-term quest to develop a true artificial intelligence, was founded in 1984 by Dr. Douglas Lenat as a lead project in the Microelectronics and Computer Technology Corporation (MCC). In 1994, Cycorp was founded to further develop, commercialize, and apply the Cyc technology. To foster the growth of semantic reasoning by the research community, Cycorp offers a no-cost license to its semantic technologies development toolkit to the research community. Additionally, it has placed the core Cyc ontology into the public domain.

Done

OpenCyc

OpenCyc Browser (knuth) - Mozilla Firefox

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http://localhost:3602/cgi-bin/cyccgi/cg?cb-start

Google

News Downloads Software Hardware Developers Help Search Shop

OpenCyc Browser (knu...)

animal Search Clear

Assert Compose Create Doc History Query

You are: [CycAdministrator](#) [Logout]
Server: knuth:3600
[Preferences](#)
[Tools](#)

Animal

[\[Create Similar\]](#) [\[Create Instance\]](#)
[\[Create Spec\]](#) [\[Rename\]](#) [\[Merge\]](#) [\[Kill\]](#)

[Documentation](#)
[Definitional Info](#)
[Internal Data](#)
[Assertions History](#)

[All Asserted Knowledge](#) (761)
[Bookkeeping Info](#) (1)
[Inferred Index](#)

[All KB Assertions](#) (760)
[All GAFs](#) (711)
▼ [Arg 1](#) (51)
▶ [isa](#) (7)
▶ [gens](#) (12)
▶ [disjointWith](#) (3)
[comment](#)
[covering](#)
[facets-Covering](#)
[facets-Generic](#) (6)
[facets-Partition](#)
[facets-Strict](#) (2)
▶ [partitionedInto](#) (5)
[prettyString](#) (5)
[nrettyString-Canonical](#)

Collection : Animal

GAF Arg : 1

Mt : **UniversalVocabularyMt**
isa : [Analyst-PertinentConcept](#) [BiologicalKingdom](#) [OrganismClassificationType](#)

Mt : **BaseKB**
isa : [ClarifyingCollectionType](#)

Mt : **BiologyMt**
isa : [KEClarifyingCollectionType](#)

Mt : **TopicMt**
isa : [OrganismByTaxonomicKingdom-Biology-Topic](#) [Biology-Topic](#)

Mt : **UniversalVocabularyMt**
gens : [Agent-NonArtifactual](#) [AnimalBLO](#) [PerceptualAgent-Embodied](#) [Organism-Whole](#)
 [SolidTangibleThing](#)

Mt : **BaseKB**
gens : [\(CollectionUnionFn](#)
[\(TheSet DeadAnimal Animal\)\)](#) [\(CollectionUnionFn](#)
[\(TheSet Person Animal\)\)](#) [\(CollectionUnionFn](#)
[\(TheSet Animal](#)
[\(GroupFn Animal\)\)\)](#) [AnimalBLO](#)

Mt : **BiologyMt**
gens : [\(CollectionDifferenceFn HumanScaleObject SpaceInAHOC\)](#)

[Update Comm:](#) Storing Only [Agenda:](#) Idle **KB:** 5018 [System:](#) 10.128401 [Learn about ResearchCyc](#)

Done

Knowledge server

```
xterm
Start time: Wed Sep 30 23:27:42 CEST 2009
Lisp implementation: Cycorp Java SubL Runtime Environment
JVM: Sun Microsystems Inc. Java HotSpot(TM) Server VM 1.6.0_03 (1.6.0_03-b05)
Current KB: 5018
Patch Level: 10.128401
Running on: knuth
OS: Linux 2.6.23.17-88.fc7 (i386)
Working directory: /home/peterl/Research/Cyc/opencyc-2.0/server/cyc/run
Total memory allocated to VM: 1348MB.
Memory currently used: 541MB.
Memory currently available: 806MB.
Initializing HL backing store caches from units/5018/.
;; At this point the cyc http server is running and you can access
;; Cyc directly via the local web browser.
;; http://localhost:3602/cgi-bin/cyccgi/cg?cb-start
;; You can browse cyc via the Guest account or perform updates by
;; logging on as CycAdminstrator.
CYC(1):
```

Representation of relations

1. Semantic nets:

- syntactic: nodes and relations between nodes
- represented as a graph
- semantics of nodes and relationships

2. Frames:

- (binary) relations represented as attributes
- inheritance
- subtyping

3. Description logic:

- concepts
- binary relationships
- restrictions

Basics of description logics

Example DL:

ALC = Attribute concept Language with Complement

Basic ingredients:

- concepts
- roles
- Boolean operators

“A man is married to a doctor, and all of whose children are either doctors or professors”

$$\text{Human} \sqcap \neg \text{Female} \sqcap \exists \text{married}.\text{Doctor} \\ \sqcap (\forall \text{hasChild}.\text{(\text{Doctor} \sqcup \text{Professor})})$$

Language elements

Concept descriptions:

- primitive **concept** C , e.g., Human, \top (most general), \perp (empty)
- primitive **role** r , e.g., hasChild
- **conjunction** \sqcap , e.g., SmartHuman \sqcap Student
- **disjunction** \sqcup , e.g., Truck \sqcup Car
- **complement** \neg , e.g., \neg Human
- **value restriction** $\forall r.C$, e.g., \forall hasChild.Doctor
- **existential restriction** $\exists r.C$, e.g., \exists happyChild.Parent

All understood in terms of **(groups of) individuals** and properties of individuals

General concept inclusion (GCI)

General concept inclusion (GCI) also called **subsumption**

For concepts C, D :

- $C \sqsubseteq D$, e.g., Professor \sqsubseteq Person
- **definition** $C \equiv D$ is the same as $C \sqsubseteq D$ and $D \sqsubseteq C$
(Not $(C \sqsubseteq D) \sqcap (D \sqsubseteq C)$, why?)
- C in $C \equiv D$ is called a **defined** concept, whereas D consists of primitive concepts, e.g.,

Father $\equiv \neg$ Female $\sqcap \exists$ hasChild.Human

Concrete descriptions

Instances of concepts or roles, called **assertions**:

- $c : C$, means that c is an **instance** of concept C , e.g.,

John : Person

- $(b, d) : r$, means that the pair of individuals (b, d) is an instance of role r , e.g.,

(John, Mary) : marriedTo

Knowledge base

Knowledge Base = **KB**

Terminology = **TBox**

Father $\equiv \neg\text{Female} \sqcap \exists\text{hasChild.Human}$
Human \sqsubseteq Animal

Concrete assertions = **ABox**

John : Father
(John, Sheila) : hasChild

KB = (TBox, ABox)

Knowledge base

KB = (TBox, ABox):

- TBox: contains **general** descriptions, definitions, subsumptions relationships (GCI)

Father $\equiv \neg$ Female $\sqcap \exists$ hasChild.Human

Human \sqsubseteq Animal

- ABox: contains description of **individuals** (instances)

John : Father

(John, Sheila) : hasChild

- Sometimes ABox = \emptyset , then only interested in general principles: **terminological reasoning**

Meaning of description logic

In terms of **set theory**

Let $\mathcal{I} = (\Delta, \cdot)$ be an **interpretation**, then

- $\top^{\mathcal{I}} = \Delta$, and $\perp^{\mathcal{I}} = \emptyset$
- each concept $C^{\mathcal{I}} \subseteq \Delta$
- each role $r^{\mathcal{I}} \subseteq \Delta \times \Delta$
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $(\exists r.C)^{\mathcal{I}} = \{c \in \Delta \mid \exists d \in C^{\mathcal{I}} \text{ with } (c, d) \in r^{\mathcal{I}}\}$
- $(\forall r.C)^{\mathcal{I}} = \{c \in \Delta \mid \forall d \in \Delta, \text{ if } (c, d) \in r^{\mathcal{I}} \text{ then } d \in C^{\mathcal{I}}\}$

where $C^{\mathcal{I}}$ and $r^{\mathcal{I}}$ are interpretations of C and r as sets

Example

$\text{Father} \equiv \neg\text{Female} \sqcap \exists\text{hasChild.Human}$

Interpretation $\mathcal{I} = (\Delta, \cdot)$, with $\Delta = \{\text{John}, \text{Sheila}\}$

- $\text{Father}^{\mathcal{I}} = \{\text{John}\} \subseteq \Delta$
- $\text{Human}^{\mathcal{I}} = \{\text{John}, \text{Sheila}\}$
- $\text{hasChild}^{\mathcal{I}} = \{(\text{John}, \text{Sheila})\}$
- $(\exists\text{hasChild.Human})^{\mathcal{I}} = \{\text{John}\}$

Meaning of subsumption

Interpretation $\mathcal{I} = (\Delta, \cdot)$, then

$$(C \sqsubseteq D)^{\mathcal{I}} = C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$

Example:

Father \sqsubseteq Human

$\Delta = \{\text{John, Sheila}\}$, then

$$\text{Father}^{\mathcal{I}} = \{\text{John}\}$$

and

$$\text{Human}^{\mathcal{I}} = \{\text{John, Sheila}\}$$

Relationship with predicate logic

Translation function τ_x (DL \rightarrow first-order predicate logic) that introduces variable x :

- $\tau_x(C) = C(x)$

- $\tau_x(C \sqcap D) = (\tau_x(C) \wedge \tau_x(D))$

- $\tau_x(C \sqcup D) = (\tau_x(C) \vee \tau_x(D))$

- $\tau_x(\exists r.C) = \exists y r(x, y) \wedge \tau_y(C)$

- $\tau_x(\forall r.C) = \forall y r(x, y) \rightarrow \tau_y(C)$

Example: $\neg\text{Female} \sqcap \exists\text{hasChild.Human}$:

$$\tau_x(\neg\text{Female} \sqcap \exists\text{hasChild.Human})$$

$$= \tau_x(\neg\text{Female}) \wedge \tau_x(\exists\text{hasChild.Human})$$

$$= \neg\text{Female}(x) \wedge \tau_x(\exists\text{hasChild.Human})$$

$$= \neg\text{Female}(x) \wedge \exists y(\text{hasChild}(x, y) \wedge \text{Human}(y))$$

Relationship with predicate logic

- GCIs $C \sqsubseteq D$ in TBox:

$$\bigwedge_{C \sqsubseteq D \in \text{TBox}} \forall x (\tau_x(C) \rightarrow \tau_x(D))$$

- Thus, \sqsubseteq becomes logical implication

Example: Translate

UnivTeacher \sqsubseteq Prof \sqcup \neg Undergraduate

to predicate logic:

$$\begin{aligned} & \tau_x(\text{UnivTeacher} \sqsubseteq \text{Prof} \sqcup \neg \text{Undergraduate}) \\ &= \forall x (\text{UnivTeacher}(x) \rightarrow \tau_x(\text{Prof} \sqcup \neg \text{Undergraduate})) \\ &= \forall x (\text{UnivTeacher}(x) \rightarrow (\text{Prof}(x) \vee \neg \text{Undergraduate}(x))) \end{aligned}$$

Relationship with predicate logic

- Translation of ABox:

$$\tau_x(\text{ABox}) = \bigwedge_{c:C \in \text{ABox}} \tau_c(C) \wedge \bigwedge_{(b,d):r \in \text{ABox}} r(b, d)$$

- Thus, unit clauses of the form $C(c)$ and $r(b, d)$

Example:

$\text{ABox} = \{\text{John} : \text{Father}, (\text{John}, \text{Sheila}) : \text{hasChild}\}$

In first-order logic:

$\text{Father}(\text{John}) \wedge \text{hasChild}(\text{John}, \text{Sheila})$

Reasoning

Possible procedure:

- Translate DL knowledge base to first-order logic
- Use a reasoning engine, e.g., **resolution**, for reasoning, then

$$\tau(\text{KB}) \vdash \phi$$

with ϕ something like $\tau_x(\text{Prof} \sqsubseteq \text{Human})$ becomes:

$$\tau(\text{KB}) \wedge \neg\phi \vdash \perp$$

(i.e., if KB is consistent and $\text{KB} \cup \neg\phi$ is inconsistent, then ϕ follows from KB)

- However, special purpose reasoning may be advantageous

Typical questions

Let $KB = (TBox, ABox)$, then:

- $KB \models C \sqsubseteq D?$ (is C subsumed by D ?)
- $KB \models c : C?$ (is c an instance of C ?)
- $KB \models (b, d) : r?$ (is the pair (b, d) true for role r ?)

Reasoning can be reduced to **consistency checking**:

- $(TBox, ABox \cup \{c : C \sqcap \neg D\}) \vdash \perp$
- $(TBox, ABox \cup \{c : \neg C\}) \vdash \perp$
- $(TBox, ABox \cup \{(b, d) : \neg r\}) \vdash \perp$

Summary DL

- Description logics: restricted logical languages for the representation of conceptual information and terminologies
- Basis for all large-scale attempts for the development of knowledge bases, e.g. semantic web and biomedical ontologies
- Advantages:
 - Restricted syntax allows developing special purpose, efficient reasoning systems (e.g., Racer, Fact, Cyc)
 - \mathcal{ALC} is decidable! (a 2 variable fragment of first-order logic)
- Disadvantage: many things cannot be represented in a DL

Frame formalism

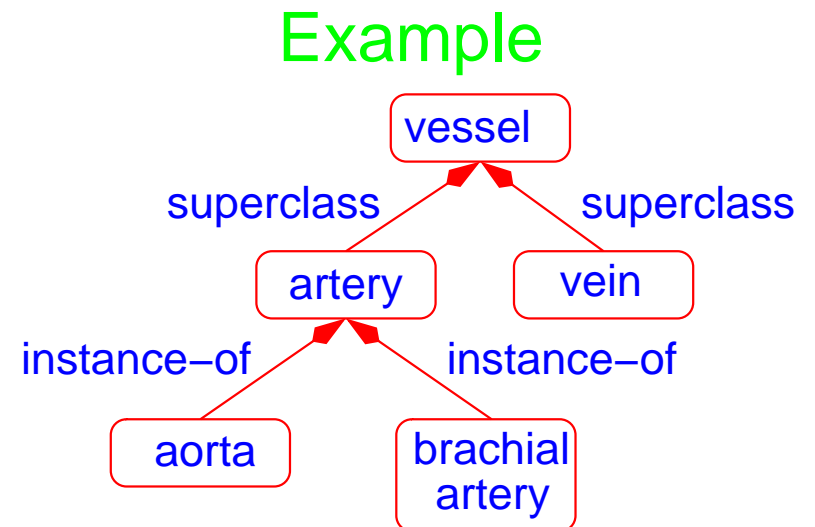
Predecessor of description logics, still in use

Concepts and **properties** of concepts

Example (human anatomy): “An artery is a vessel. An artery transports blood from the heart to the tissues, and is characterised by a thick wall and much muscular tissue.”

Frame taxonomy: hierarchical organisation

- **nodes** (vertices): classes and instances
- **arcs** (arrows): superclasses and instance-of relationships



Syntax

Basic elements:

- **subclass relationship** between frames
- **attributes** or **slots**
- **values** or **fillers**

⇒ attribute-value pair or slot-filler combination

Example: (structure, tube) is an attribute-value pair

```
class vessel is  
    superclass nil;  
    structure = tube;  
    contains = blood  
end
```

```
class artery is  
    superclass vessel;  
    wall = muscular  
end
```

```
instance aorta is  
    instance-of artery;  
    diameter = 3cm  
end
```

Meaning of frames

Semantics in terms of **predicate logic**:

class C **is**

superclass S ;

$a_1 = b_1$;

\vdots

$a_n = b_n$

end

$$\forall x(C(x) \rightarrow S(x))$$

$$\forall x(C(x) \rightarrow a_1(x) = b_1)$$

\vdots

$$\forall x(C(x) \rightarrow a_n(x) = b_n)$$

instance i **is**

instance-of C ;

$d_1 = e_1$;

\vdots

$d_m = e_m$

end

$$C(i)$$

$$d_1(i) = e_1$$

\vdots

$$d_m(i) = e_m$$

Relationship with description logic

```
class  $C$  is  
  superclass  $S$ ;  
   $a_1 = b_1$ ;  
   $\vdots$   
   $a_n = b_n$   
end
```

```
instance  $i$  is  
  instance-of  $C$ ;  
   $d_1 = e_1$ ;  
   $\vdots$   
   $d_m = e_m$   
end
```

TBox:

$$C \sqsubseteq S$$

$$C \sqsubseteq \exists a_1.\{b_1\}$$

\vdots

$$C \sqsubseteq \exists a_n.\{b_n\}$$

($\{i\}$ is a concept if i is an individual)

ABox:

$$i : C$$

$$(i, e_1) : d_1$$

\vdots

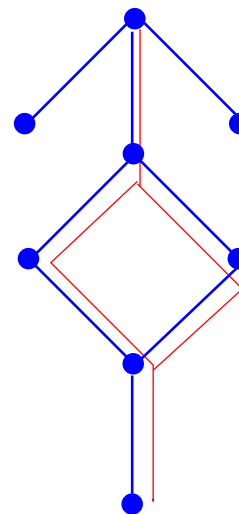
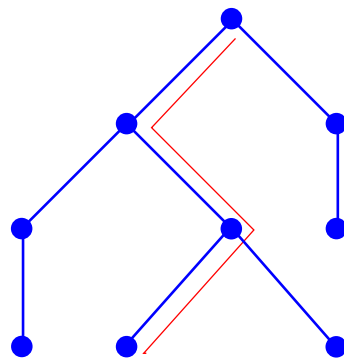
$$(i, e_m) : d_m$$

Reasoning

- Basic reasoning method is called **inheritance**, sometimes example of **non-monotonic** reasoning
- Frame inherits attribute-value pairs from its generalisations

Two types of inheritance:

1. **single**: tree-shaped taxonomy
2. **multiple**: general graph-shaped taxonomy (loops)

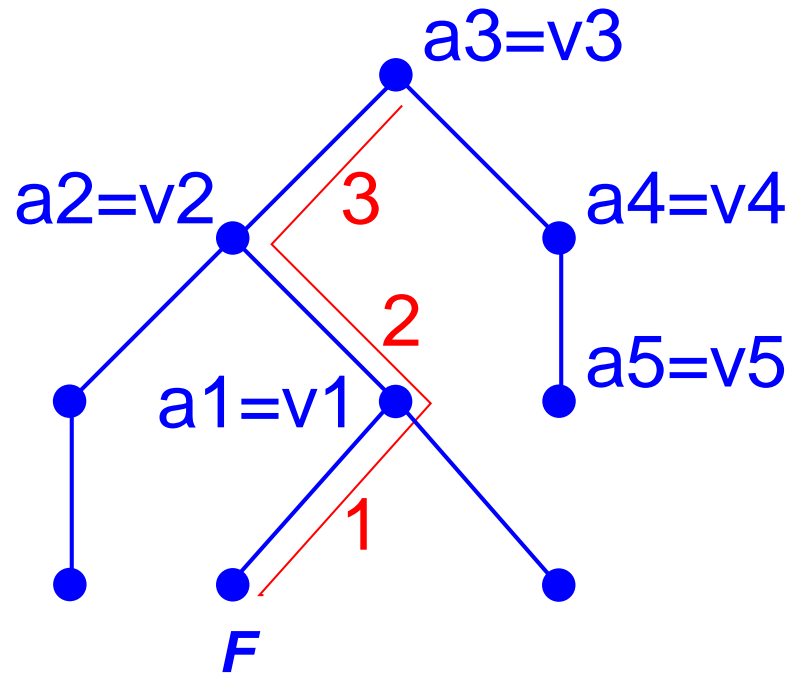


Single inheritance

Requirement: unique attribute names

Example

```
function Inherit(frame, a-v-pairs)
  if frame = nil
  then return(a-v-pairs)
  fi;
  a-v-pairs ← a-v-pairs ∪
    AttributePart(frame);
  return(Inherit(superframe(frame),
    a-v-pairs))
end
```



F inherits $a_1 = v_1, a_2 = v_2$ en
 $a_3 = v_3$

Exceptions

Non-unique attribute names \Rightarrow exceptions / inconsistency

Example

```
class artery is  
  instance-of artery;  
  superclass vessel;  
  blood = oxygenrich  
end
```

```
instance pulmonary-a is  
  instance-of artery;  
  blood = oxygenpoor  
end
```

$\forall x(\text{artery}(x) \rightarrow \text{vessel}(x))$

$\forall x(\text{artery}(x) \rightarrow \text{blood}(x) = \text{oxygenrich})$

$\text{artery}(\text{pulmonary-a})$

$\text{blood}(\text{pulmonary-a}) = \text{oxygenpoor}$

Logically inconsistent!

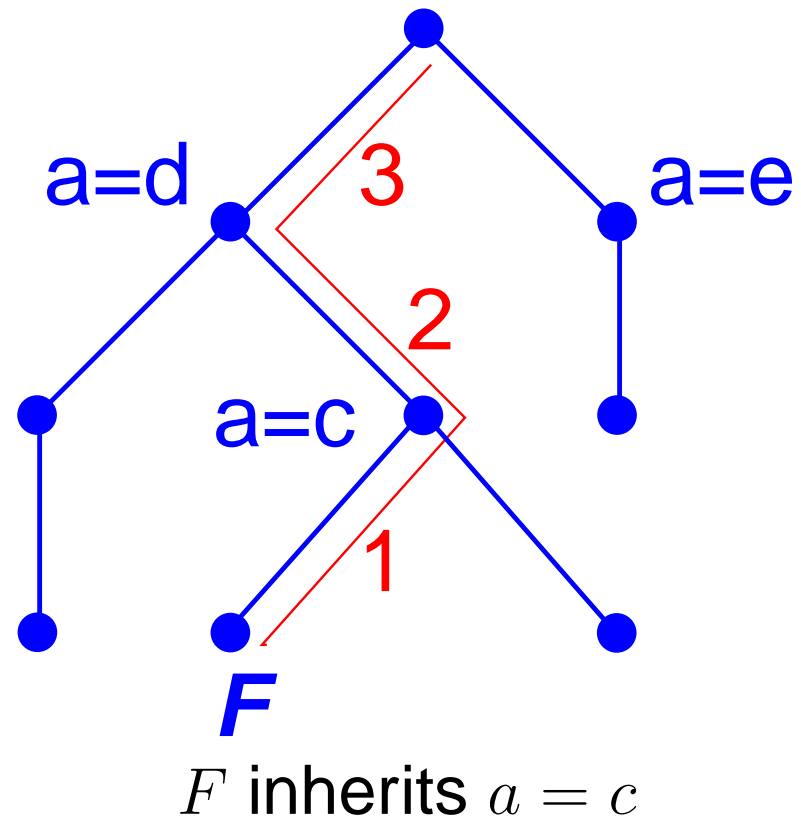
Solution: local 'overwriting'

Single inheritance with exceptions

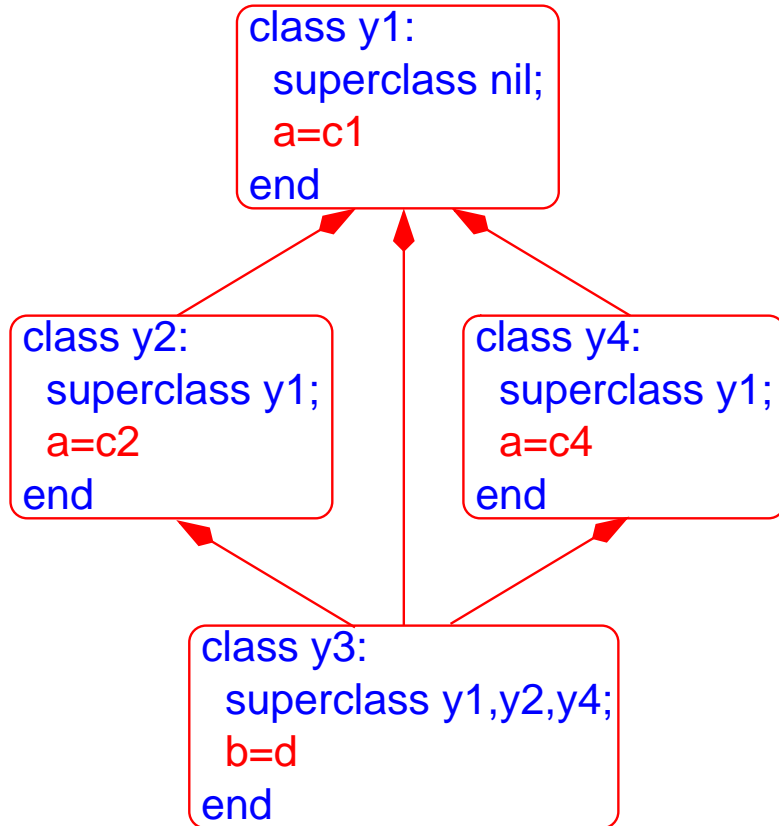
Search the taxonomy until a value is found:

Example

```
function Inherit(frame, a-v-pairs)
  if frame = nil
  then return(a-v-pairs)
  fi;
  pairs ← AttributePart(frame);
  a-v-pairs ← a-v-pairs ∪
    NewAttributes(pairs, a-v-pairs);
  return(Inherit(superframe(frame), a-v-pairs))
end
```



Multiple inheritance with exceptions



Questions:

1. Taxonomy **consistent**?
2. If yes, **what are the inherited values** for attribute *a* vof *y3*?

Requirement: solution must work for both: **graph** and **tree** shaped taxonomies!

Monotonic reasoning

- Inheritance with exceptions example of **non-monotonic reasoning**
- **Monotonic** reasoning:
 - knowledge base KB
 - **add** knowledge to KB and obtain new knowledge base KB'
 - if $KB \vdash \text{Results}$ and $KB' \vdash \text{Results}'$ then $\text{Results} \subseteq \text{Results}'$
 - **more knowledge yields more results**

- Example:

$$KB = \{P \rightarrow Q\}$$

$$\text{Results} = KB$$

$$KB' = \{P \rightarrow Q, P\}$$

$$\text{Results}' = KB' \cup \{Q\} = KB \cup \{P, Q\}$$

Non-monotonic reasoning

- Non-monotonic reasoning is more close related to human reasoning
 - knowledge base KB
 - **add** knowledge to KB and obtain new knowledge base KB'
 - if $KB \vdash \text{Results}$ and $KB' \vdash \text{Results}'$ then $\text{Results} \subseteq \text{Results}'$ does not hold in general
 - **more knowledge does not necessarily yield more results**
- Example (\vdash is non-monotonic):
 - $\{ \text{artery}(\text{left-pulmonary-artery}),$
 - $\text{blood}(\text{left-pulmonary-artery}) = \text{oxygen-poor},$
 - $\forall x(\text{artery}(x) \rightarrow \text{blood}(x) = \text{oxygen-rich}) \}$
 - $\vdash \text{blood}(\text{left-pulmonary-artery}) = \text{oxygen-poor}$