Exercises Model-based Reasoning

Exercise 1

Consider the diagnostic problem DP with system description SD, which contains the following generic descriptions of components:

$\forall x(\mathrm{ANDG}(x) \rightarrow \mathrm{and}(\mathrm{in}_1(x), \mathrm{in}_2(x)) = \mathrm{out}(x))$
$\forall x(\mathrm{ORG}(x) \rightarrow \mathrm{or}(\mathrm{in}_1(x), \mathrm{in}_2(x)) = \mathrm{out}(x))$
$\forall x((\mathrm{INV}(x) \rightarrow \neg(\mathrm{in}(x) = \mathrm{out}(x)))$

a. Show how this system description should be adapted to employ it for consistency-based diagnosis.

b. What is the role of the system description in model-based diagnosis?

c. Design a circuit consisting of two ANDG components, one INV component, and one ORG component, with appropriate wiring, inputs and outputs.

d. Give an example showing how this circuits converts inputs into outputs, i.e., give a set of observations OBS that match the expected, normal behaviour of the circuit.

e. Add to SD (a) logical formula(s) describing the behaviour of a wire(s). You may need more than one formula; try to understand why.

Exercise 2

The concept of hitting sets underlies the algorithm of consistency-based diagnosis, i.e., to determine diagnoses for a diagnostic problem. As explained at the lecture, diagnoses for a system are exactly the (minimal) hitting sets of the system's conflict sets. In this exercise we intend to take a closer look at determining hitting sets from a set of conflict sets. Now, let $F = \{\{5,3,4\},\{2,4,6\},\{1,3,7\},\{3\}\}$ be a set of sets.

a. Read the slides on hitting sets and in particular the definition of the notion of hitting set.

   Now, determine all the hitting sets of the given set F.

b. Which of these hitting sets are minimal (by minimal we mean minimal according to the subset relationship $\subseteq$)?

c. Finally, build a hitting-set tree $T = (V, E, L_V, l_E)$, with set of nodes V and edges (branches) $E \subseteq V \times V$, and two label functions:

$$l_V : V \rightarrow F \cup \{\checkmark\},$$

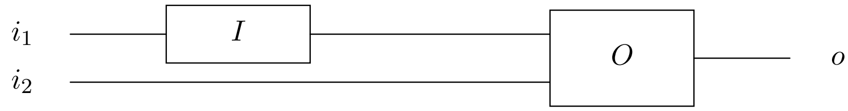
   which labels each node in V with a set S from the set F or a tick ($\checkmark$), and

$$l_E : E \rightarrow \bigcup_{S \in F} S,$$

   which labels each edge in E with an element coming from an element in a set $S \in F$. Note that there are multiple ways to do this.

Exercise 3

Consider the schema of a circuit shown in the following figure:



where

- I implements an inverter, INV(I), with the following behaviour:

  $$\forall x((\mathrm{INV}(x) \wedge \neg \mathrm{Ab}(x)) \rightarrow (out(x) = 1 \leftrightarrow in(x) = 0))$$

  (Note the difference in specification with the inverter in Exercise 1; is there a major difference between the two specifications?)

- O implements a logical OR gate, ORG(O), with the following behaviour:

  $$\forall x((\mathrm{ORG}(x) \wedge \neg \mathrm{Ab}(x)) \rightarrow out(x) = or(in_1(x), in_2(x)))$$

The following observations have been made $\mathrm{OBS} = \{i_1 = 0, i_2 = 0, o = 0\}$.

a. Give the system specification SYS of the circuit.

b. Determine the diagnoses according to the theory of consistency-based diagnosis.

c. Determine all conflict sets, and next all minimal hitting sets. Compare your results with those to question 3.b.

Exercise 4

a. Consider the diagnostic problem $\mathrm{DP} = (\mathrm{SYS}, \mathrm{OBS})$ in the theory of consistency-based diagnosis, with:

- $\mathrm{SYS} = (\mathrm{SD}, \mathrm{COMPS})$ is the system specification with
  - system description

    $$\begin{aligned}\mathrm{SD} = \{&\forall x((\mathrm{ANDG}(x) \wedge \neg \mathrm{Ab}(x)) \rightarrow (and(in(1,x), in(2,x)) = out(x))),\\ &\forall x((\mathrm{ORG}(x) \wedge \neg \mathrm{Ab}(x)) \rightarrow (or(in(1,x), in(2,x)) = out(x))),\\ &in(2, O_1) = in(2, O_2),\\ &in(2, A_2) = in(2, A_1),\\ &out(O_1) = in(1, A_1),\\ &out(O_1) = in(1, A_2),\\ &out(A_2) = in(1, O_2),\\ &\mathrm{ORG}(O_1), \mathrm{ORG}(O_2), \mathrm{ANDG}(A_1), \mathrm{ANDG}(A_2)\};\end{aligned}$$

  - $\mathrm{COMPS} = \{A_1, A_2, O_1, O_2\}$;
- $\mathrm{OBS} = \{in(1, O_1) = 1, in(2, O_1) = 0, in(2, A_2) = 1, out(A_1) = 0, out(O_2) = 0\}$ is the set of observations

Determine the set of conflict sets of DP. Which conflict sets are minimal? Determine the set of diagnoses with the hitting-set algorithm.

b. Give an example of an optimisation of the hitting-set algorithm that reduces the size of the hitting-set tree.

Exercise 5

Consistency-based diagnosis is frequently characterised as a form of non-monotonic reasoning. Give an simple example showing that this characterisation is right.

Exercise 6

a. Describe a problem situation where consistency-based diagnosis is the only possible diagnostic method. Explain your answer.

b. Formalisation of the notion of causality in abductive diagnosis is usually done by means of logical implication $\rightarrow$. Which properties of logical implication fit the intuitive meaning of causality well? Which property of logical implication does not fit?

Exercise 7

Consider the causal specification $\Sigma = (\Delta, \Phi, \mathcal{R})$, where

- $\Delta = \{d_1, d_2, d_3, \alpha_1, \alpha_2\}$ is a set of defects (d_1, d_2, d_3) and assumption literals (α_1, α_2);
- $\Phi = \{f_1, f_2, f_3\}$ is a set of observables;
- $\mathcal{R} = \{d_1 \wedge \alpha_1 \rightarrow d_2,$
  $d_1 \rightarrow f_1,$
  $d_2 \wedge \alpha_2 \rightarrow f_2,$
  $d_2 \wedge d_3 \rightarrow f_3\}$

  is a model of abnormal behaviour.

Furthermore, let $\mathcal{P} = (\Sigma, E)$ be a diagnostic problem, with $E = \{f_1, f_3\}$ a set of observed facts.

a. Determine all abductive diagnoses for $\mathcal{P}$.

b. Determine subsequently the solution disjunction that follows from the predicate completion of $\mathcal{R}$ and the set of observed facts E under the assumption that all elements in Δ, with the exception of d_2, are abducible.

c. Finally, discuss the relationship between the solution formula and the abductive diagnoses for $\mathcal{P}$.