Radbound University

Security of Encryption Modes

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Spring School on Symmetric Cryptography

March 13, 2025

Keyed Symmetric Cryptography

General Setting

$$A^{K} \longleftrightarrow B^{K}$$

- Two parties, Alice and Bob, communicate over a public channel
    - They have agreed on a joint key K and use it to transmit data

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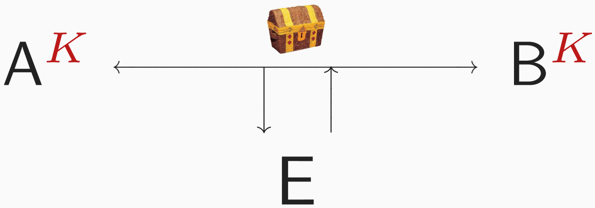

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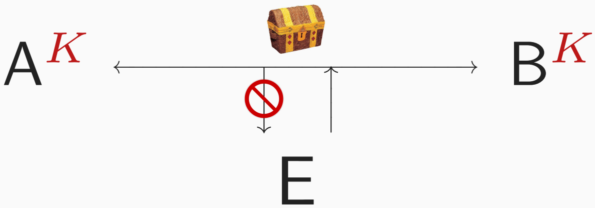

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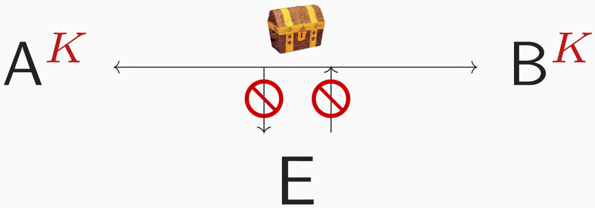

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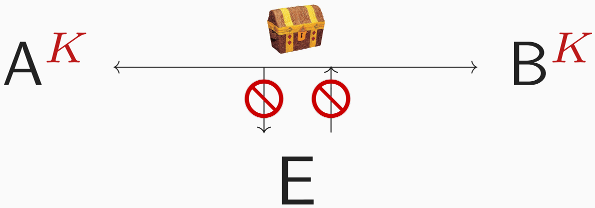

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In this presentation I will mainly focus on confidentiality

One-Time Pad Encryption

Encryption:

$$
\begin{array}{rccccccccccccccc}
M = & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
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Stream Ciphers

- Generate long keystream Z from short key K
- Much more practical!
- Security degrades:
  1. Key guessing still succeeds with probability $1/2^{|K|}$ but now with shorter key
  2. The stream cipher mechanism is another focal point of attack

Stream Cipher: Vigenère ($\approx$ 1553, Wikipedia)

$K \longrightarrow$ the Pritt stream cipher $\longrightarrow Z = K \| K \| K \| \cdots$

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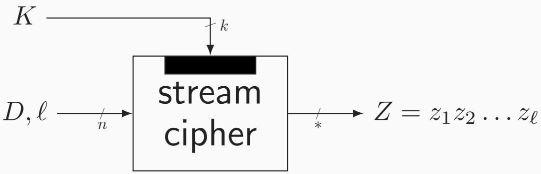
We need something more sophisticated!

How to Model Security?

Modern Stream Ciphers



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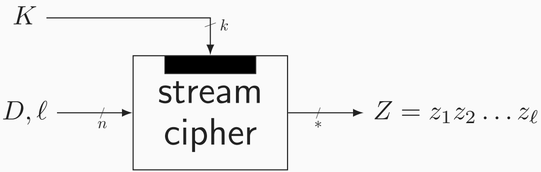

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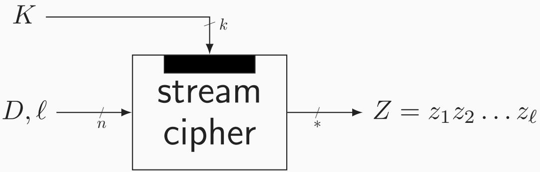

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- Example: data streams, e.g., pay TV and telephone, often split data in relatively short, numbered, frames. The frame number may serve as diversifier:

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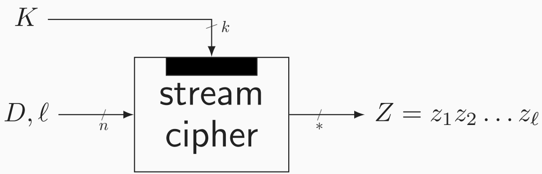

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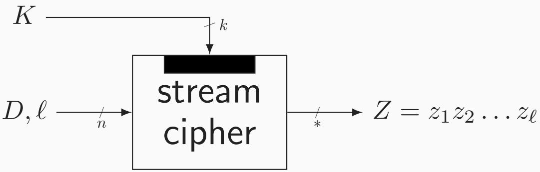
When is a stream cipher strong enough?

Stream Cipher Security, Intuition (1/3)



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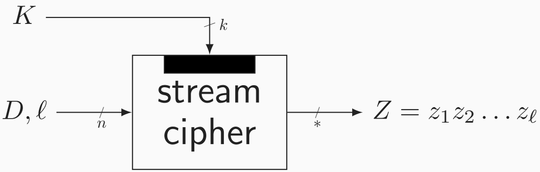

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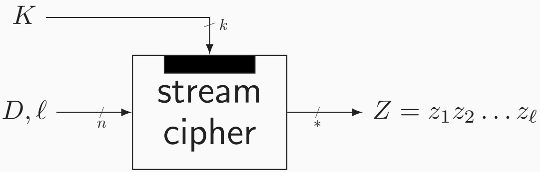

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- SC_K should behave like a random oracle

Random Oracle

- A database of input-output tuples
- Initially empty

D	Z
$\ldots$	$\ldots$
$\ldots$	$\ldots$
$\ldots$	$\ldots$
$\ldots$	$\ldots$

Intermezzo: Random Oracle

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    - update (D, Z) in the list

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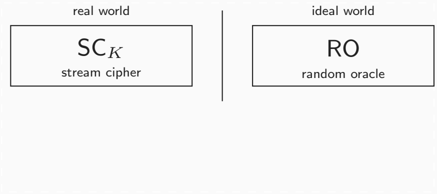
Stream Cipher Security, Intuition (2/3)



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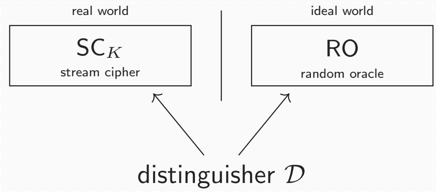

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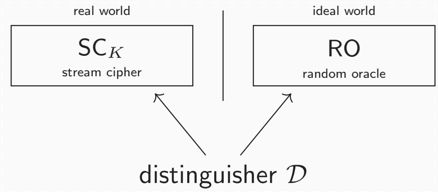

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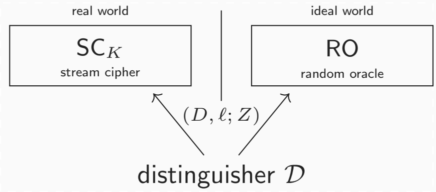

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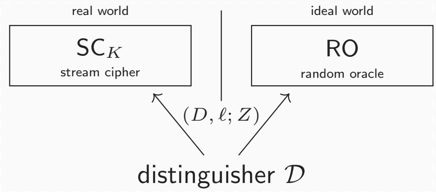

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  - At the end, $\mathcal{D}$ has to guess the outcome of the toss coin (head/tail)

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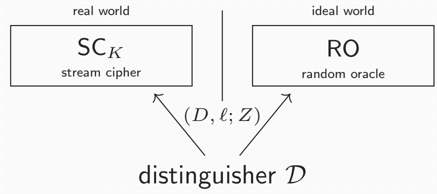

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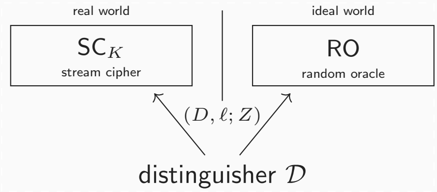

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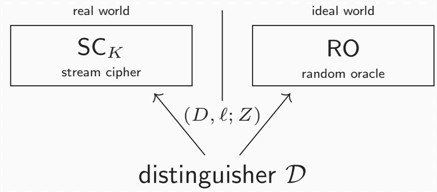

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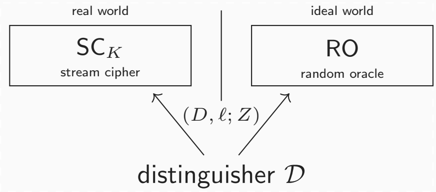

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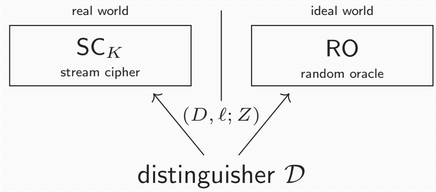

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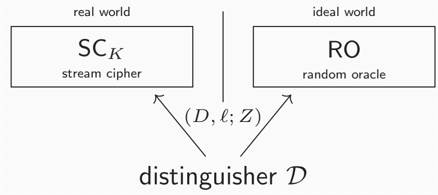
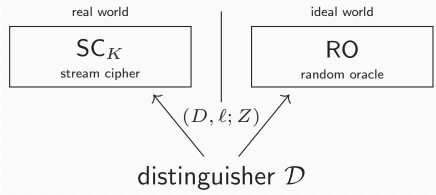
- $\mathcal{D}$ is limited by certain constraints
  - Data (or online) complexity q: total cost of queries $\mathcal{D}$ can make
  - Computation (or time) complexity t: everything that $\mathcal{D}$ can do "on its own"

Stream Cipher Security, Formal



- Two oracles: SC$_K$ (for secret key K) and RO (secret)

Stream Cipher Security, Formal



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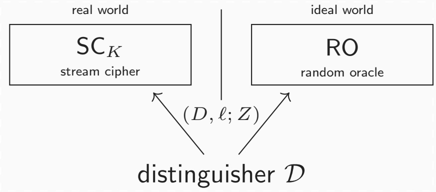

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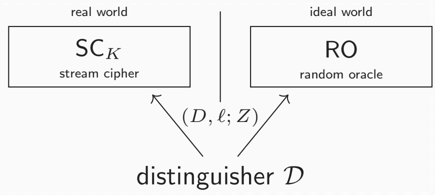

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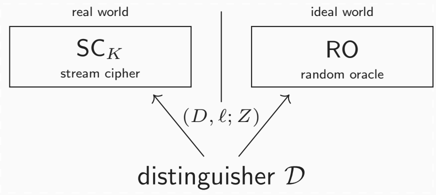

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Generic Stream Cipher Design

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- Modern approach: building construction from smaller cryptographic primitive

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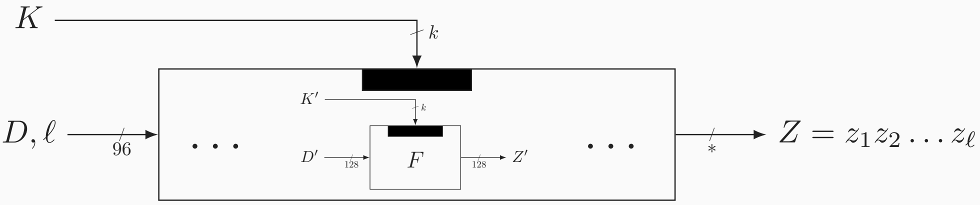
- Suppose (for the sake of argument):
  - we **know** how to build a strong stream cipher F with fixed-length output
  - we **want** to build a stream cipher with variable-length output



Design

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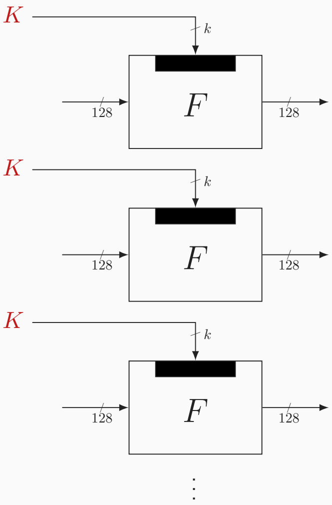

Generic Stream Cipher Design (2/2)

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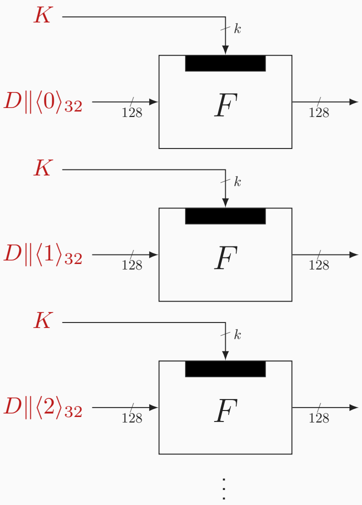

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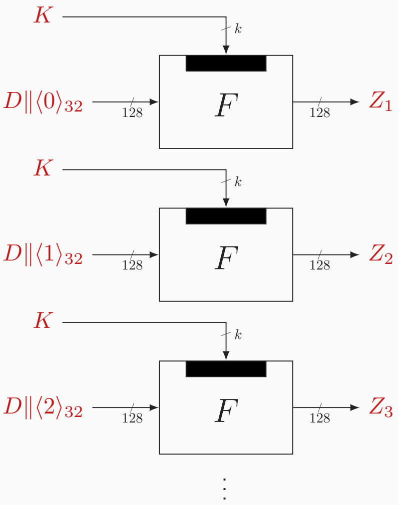
$$Z = Z_1 \parallel Z_2 \parallel Z_3 \parallel \cdots$$



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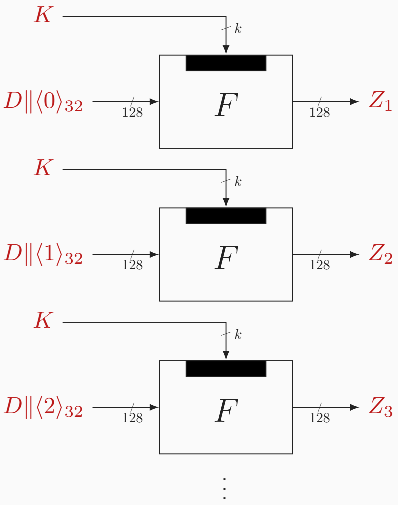

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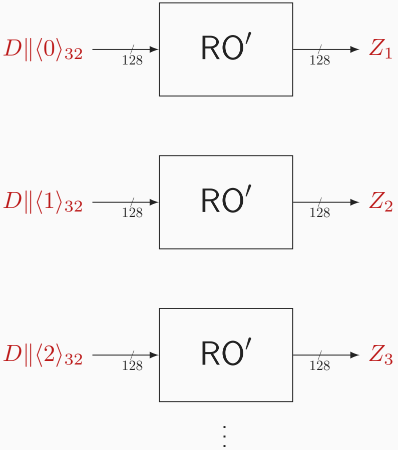

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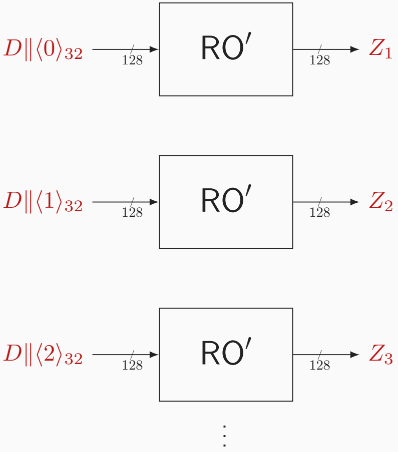

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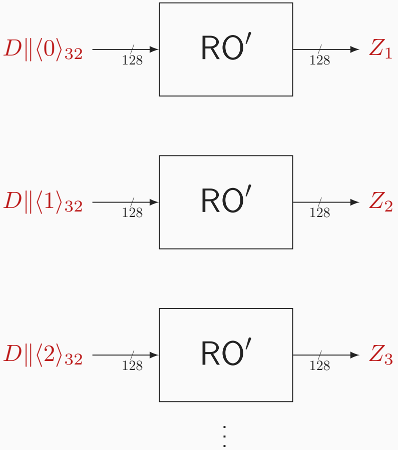

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- Evaluate primitive as
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- Concatenate outputs:

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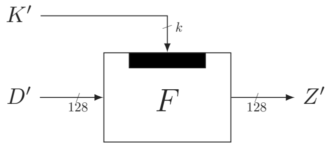
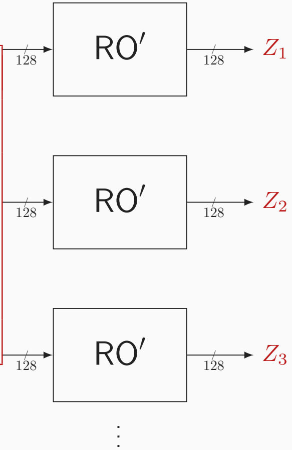
Unfortunately, we do not know how to easily construct a function



that behaves like a RO′



Block Ciphers

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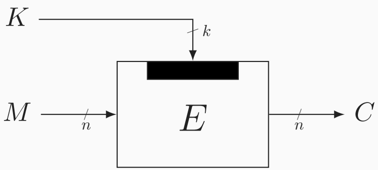

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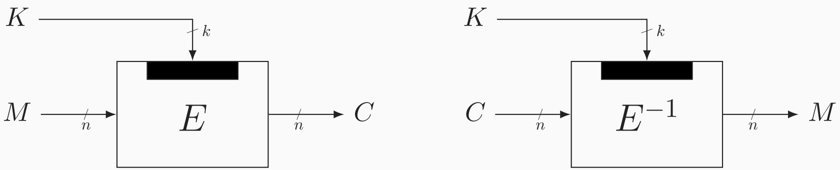

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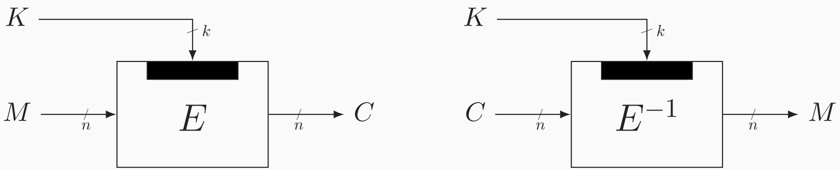

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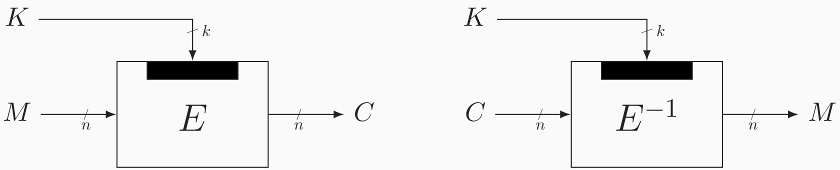

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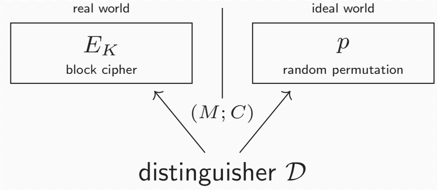
- A good block cipher should behave like a random permutation

Block Cipher Security



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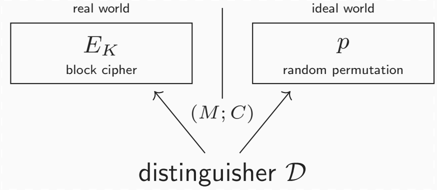

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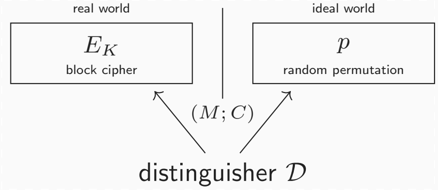

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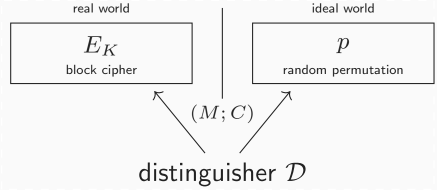

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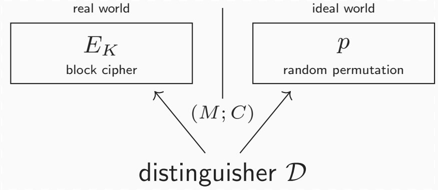

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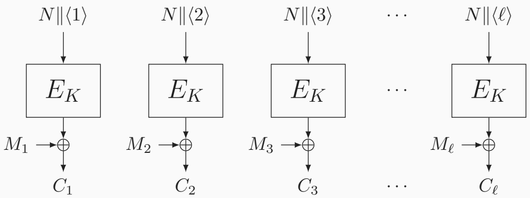
Counter Mode Encryption

Counter (CTR) Mode



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- Stream-based encryption mode
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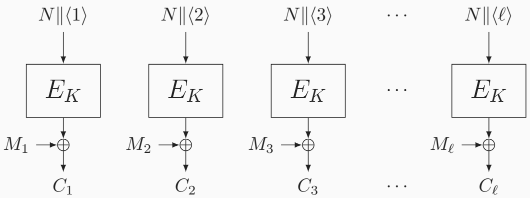

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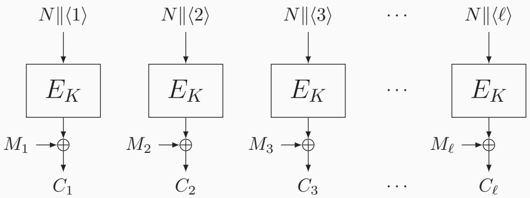

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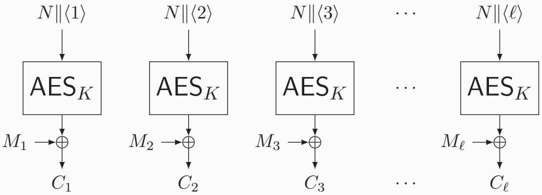
- Let us investigate that!

Security of Counter Mode Based on AES

- Let us consider counter mode based on AES: CTR[AES_K]



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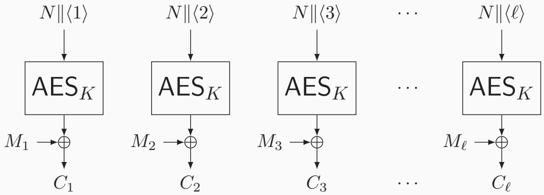

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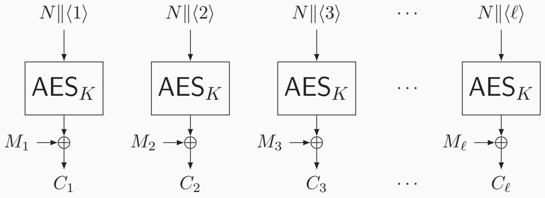

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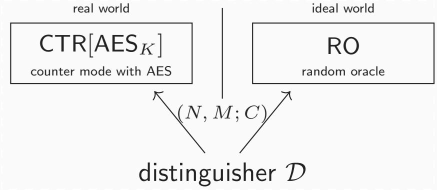
  - AES itself is sufficiently secure: $\mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q, t)$ is small

Security of Counter Mode Based on AES: Model



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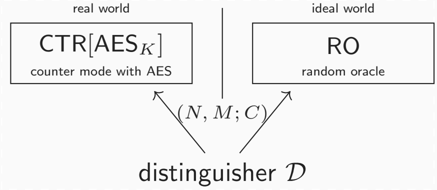

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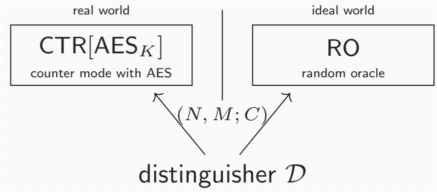

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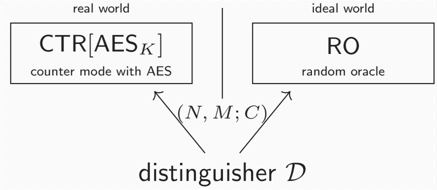

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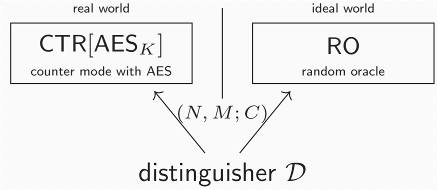

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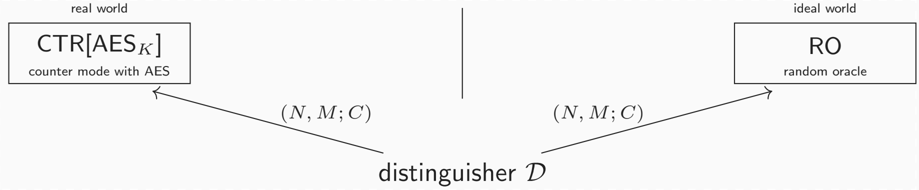

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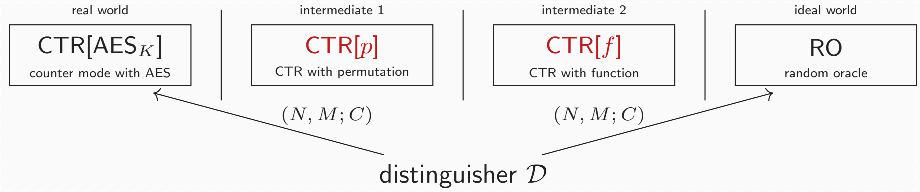

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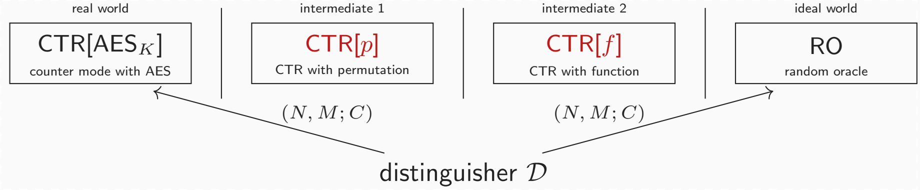

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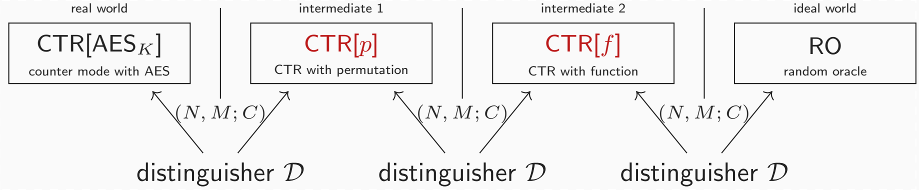

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- $\mathcal{D}'$ simulates the oracles of $\mathcal{D}$:
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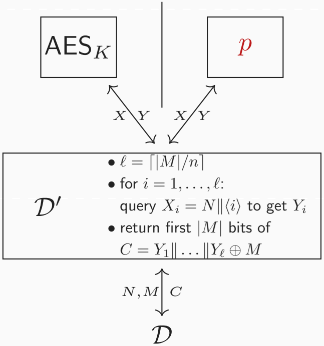

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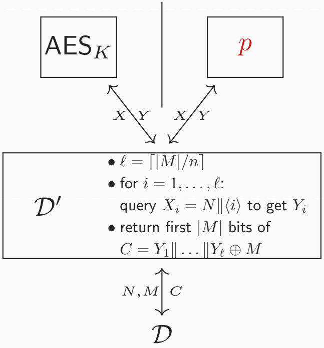

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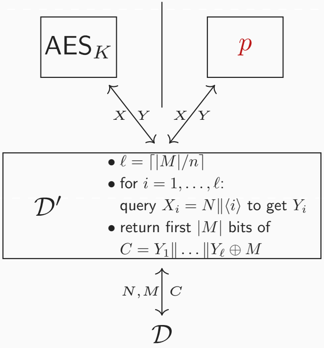
- But we have seen this distance before:
  $\Delta_{\mathcal{D}'}\left(\mathsf{AES}_K\ ;\ p\right) = \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(\mathcal{D}') \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q, t')$
  (t' slightly larger than t)



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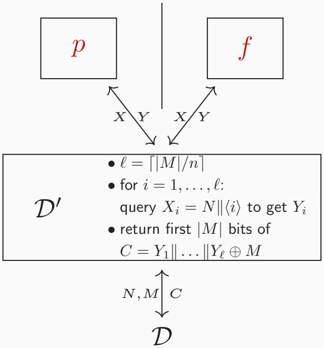

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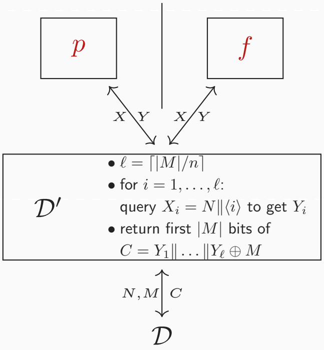

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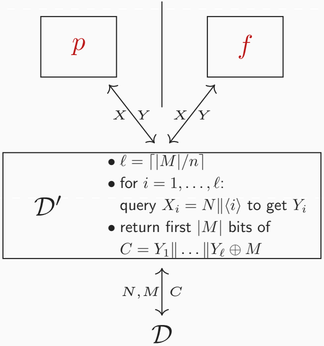
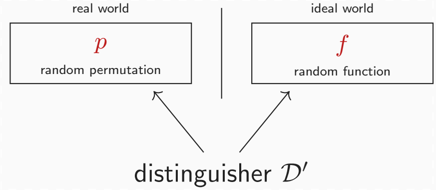
$$\Delta_{\mathcal{D}}(\text{CTR}[p]\ ;\ \text{CTR}[f]) \leq \Delta_{\mathcal{D}'}(p\ ;\ f)$$

- This is a well-known distance, called the RP-RF switch



Proof: From CTR$[p]$ to CTR$[f]$ (2/2)



- Distinguisher $\mathcal{D}'$ gets q random n-bit samples:
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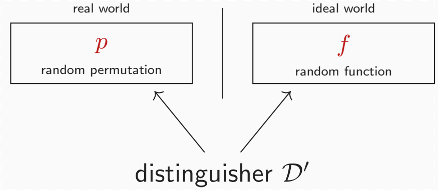
  - ideal world: with replacement



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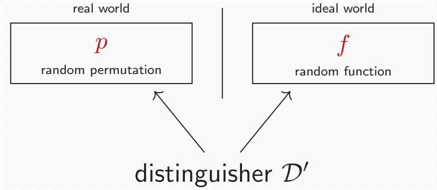

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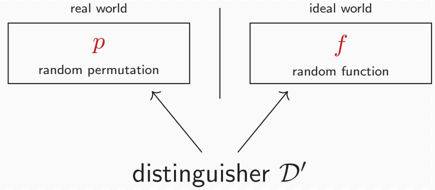

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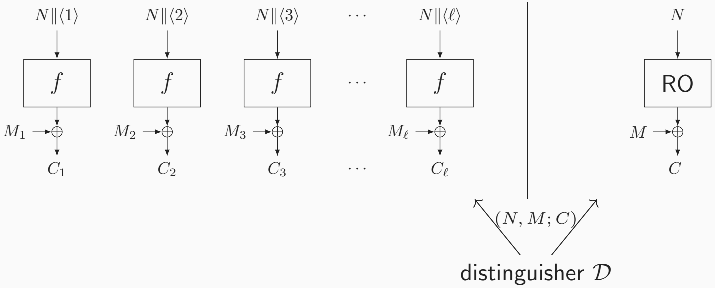
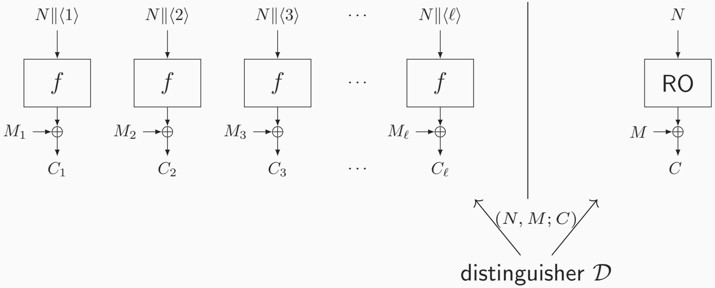
- Hence: $\Delta_{\mathcal{D}'}(p\ ;\ f) \leq \binom{q}{2}/2^n$



- In real world: f is a random function that is never evaluated for repeated $N\|\langle i\rangle$
- In ideal world: RO is a random oracle that is never evaluated for repeated N

Proof: From CTR$[f]$ to RO



- In real world: f is a random function that is never evaluated for repeated $N\|\langle i\rangle$
- In ideal world: RO is a random oracle that is never evaluated for repeated N
- Hence: $\Delta_{\mathcal{D}}\left(\text{CTR}[f]\ ;\ \text{RO}\right) = 0$

Proof: Conclusion

- Recall goal: bounding $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D})$ for any $\mathcal{D}$ querying q blocks in t time

Proof: Conclusion

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- From the triangle inequality and bounds on the three individual terms:

$$\begin{aligned}
\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D}) &= \Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\ ;\ \mathsf{RO}\right) \\
&\leq \Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\ ;\ \mathsf{CTR}[p]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[p]\ ;\ \mathsf{CTR}[f]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[f]\ ;\ \mathsf{RO}\right) \\
&\leq \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q, t') + \binom{q}{2}/2^n + 0
\end{aligned}$$

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- As this reasoning holds for all distinguishers $\mathcal{D}$ querying q blocks in t time, we obtain:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q,t') + \binom{q}{2}/2^n$$

Beyond Birthday Bound Security

Birthday Paradox

For a random selection of 23 people, with a probability at least 50% two of them share the same birthday

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General Birthday Paradox

- Consider space $\mathcal{S} = \{0,1\}^n$
- Randomly draw q elements from $\mathcal{S}$
- Expected number of collisions:

$$\mathbf{Ex}\left[\text{collisions}\right] = \binom{q}{2}/2^n$$

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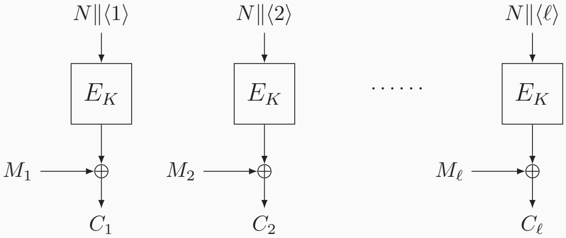
$$\mathbf{Ex}\left[\text{collisions}\right] = \binom{q}{2} / 2^n$$

- Important phenomenon in cryptography

Counter Mode Based on Pseudorandom Permutation



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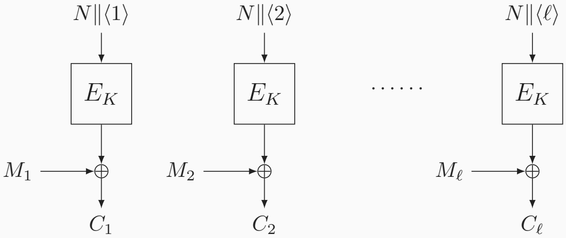
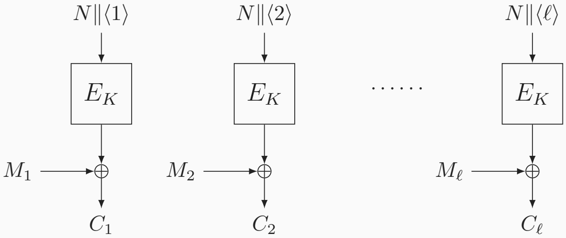


- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(q,t') + \binom{q}{2}/2^n$$

Counter Mode Based on Pseudorandom Permutation

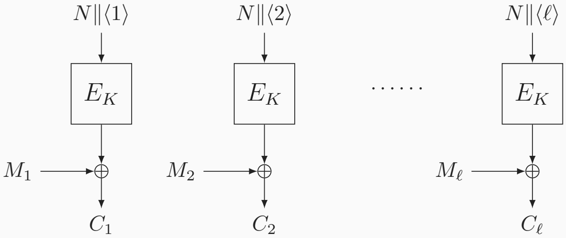


- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(q,t') + \binom{q}{2}/2^n$$

- $\mathsf{CTR}[E]$ is secure as long as:
  - E_K is a secure PRP
  - Number of encrypted blocks $q \ll 2^{n/2}$

Counter Mode Based on Pseudorandom Permutation



- $M_i \oplus C_i$ is distinct for all q blocks
- Unlikely to happen for random string

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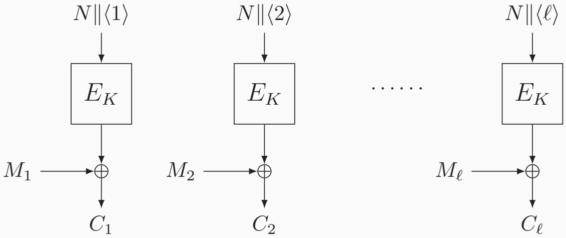
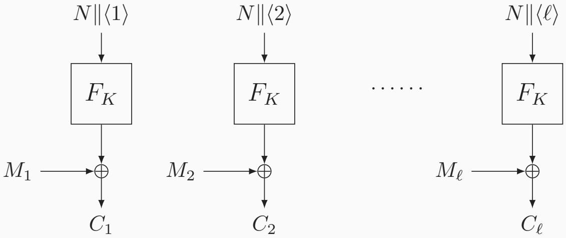


- $M_i \oplus C_i$ is distinct for all q blocks
- Unlikely to happen for random string
- Distinguishing attack in $q \approx 2^{n/2}$ blocks:

$$\binom{q}{2}/2^n \lesssim \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t)$$

Counter Mode Based on Pseudorandom Function



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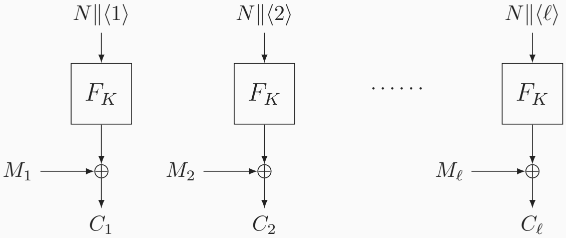
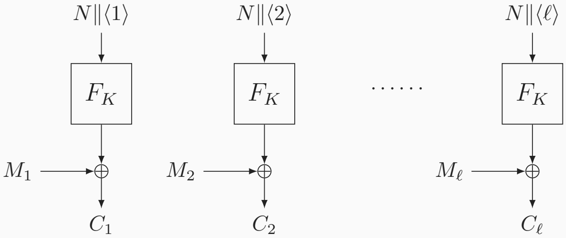


- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[F]}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(q,t')$$

Counter Mode Based on Pseudorandom Function



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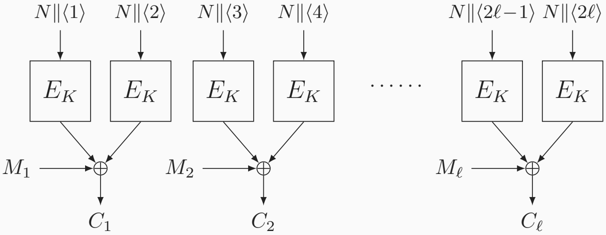
- $\mathsf{CTR}[F]$ is secure as long as F_K is a secure PRF
- Birthday bound security loss disappeared

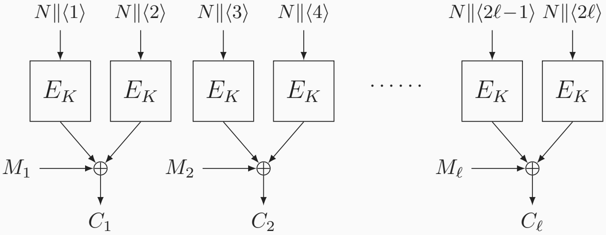
Counter Mode Based on XoP



- Security bound [Pat08a, DHT17]:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[XoP]}}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{XoP}}(q,t')$$

Counter Mode Based on XoP



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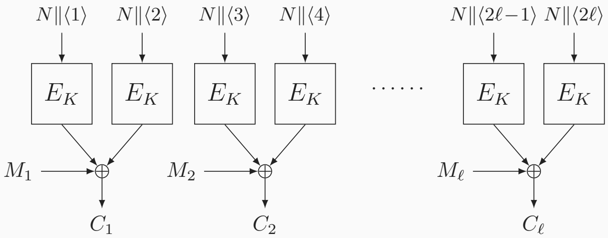
Counter Mode Based on XoP



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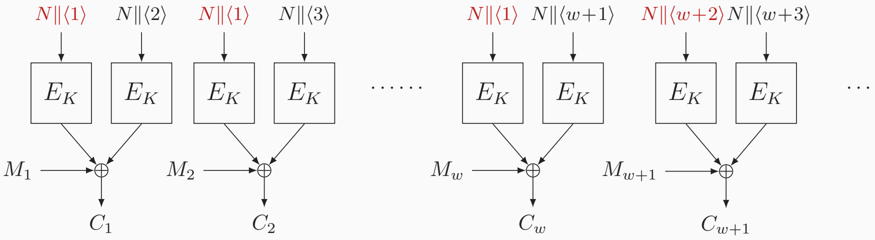
- Beyond birthday bound but 2x as expensive as CTR[E]

CENC by Iwata [Iwa06]



- One subkey used for $w \geq 1$ encryptions

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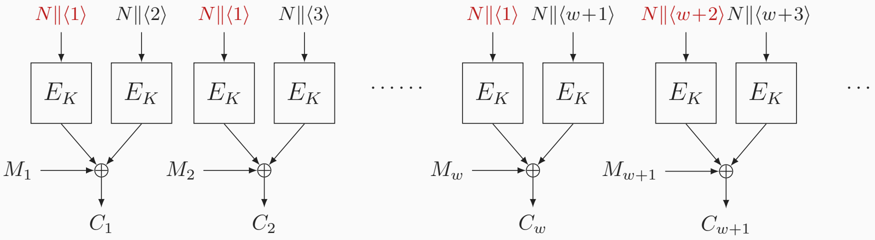

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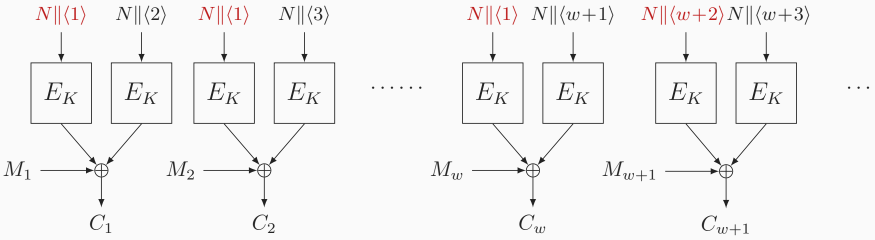

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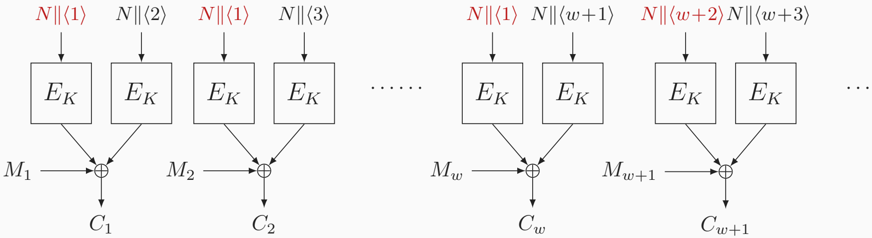

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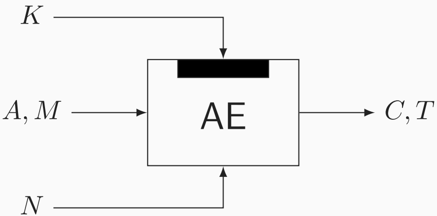
- Security of XoP and XoP$[w]$ can be proven using mirror theory [Pat03]

Authenticated Encryption and GCM

Authenticated Encryption



- Using key K:
  - Message M is encrypted in ciphertext C
  - Associated data A and message M are authenticated using T

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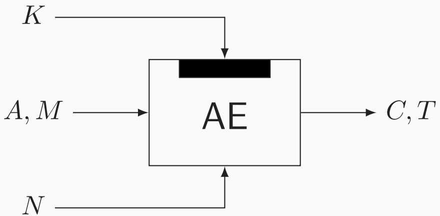

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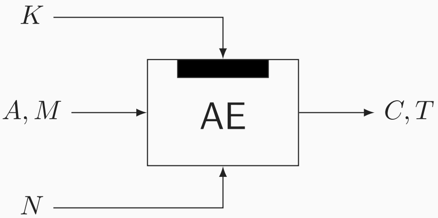
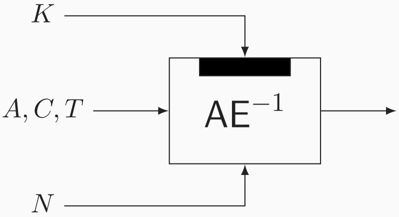


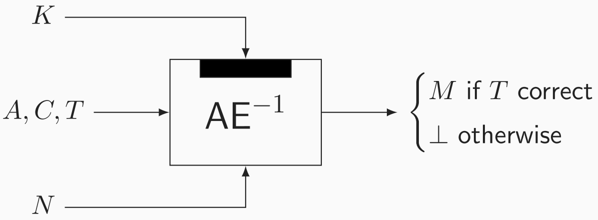
- Using key K:
  - Message M is encrypted in ciphertext C
  - Associated data A and message M are authenticated using T
- Nonce N randomizes the scheme
- Key, nonce, and tag are typically of fixed size
- Associated data, message, and ciphertext could be arbitrary length

Authenticated Decryption



- Authenticated decryption needs to satisfy that
  - Message disclosed if tag is correct
  - Message is not leaked if tag is incorrect

Authenticated Decryption



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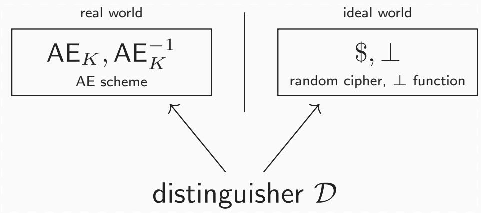
Authenticated Encryption Security



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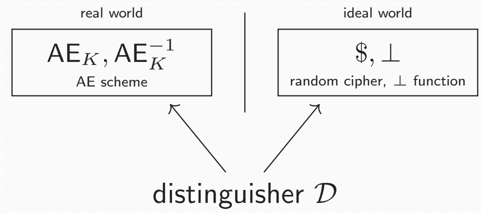
Authenticated Encryption Security



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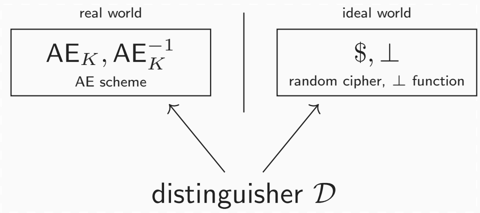

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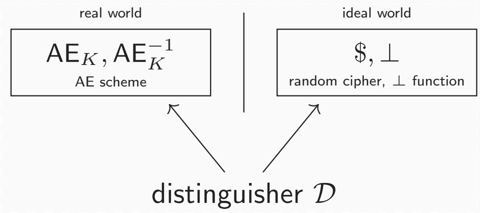
  $\to$ unique nonce for each encryption query, and no trivial queries
- $\mathcal{D}$ tries to determine which oracle it communicates with

Authenticated Encryption Security



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  $\rightarrow$ unique nonce for each encryption query, and no trivial queries
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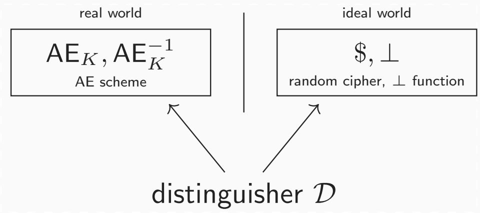
$$\mathbf{Adv}_{\mathsf{AE}}^{\mathrm{ae}}(\mathcal{D}) = \Delta_{\mathcal{D}}\left(\mathsf{AE}_K, \mathsf{AE}_K^{-1} \; ; \; \$, \perp\right) = \left|\mathbf{Pr}\left(\mathcal{D}^{\mathsf{AE}_K, \mathsf{AE}_K^{-1}} = 1\right) - \mathbf{Pr}\left(\mathcal{D}^{\$, \perp} = 1\right)\right|$$

Authenticated Encryption Security



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- $\mathbf{Adv}^{\mathrm{ae}}_{\mathsf{AE}}(q_e, q_v)$: maximum advantage over any $\mathcal{D}$ with query complexity q_e, q_v

Universal Hash Functions

- Consider hash function $H : \{0,1\}^k \times \{0,1\}^* \to \{0,1\}^t$
- H is ε-XOR-universal if $\mathbf{Pr}_K\left(H_K(M) \oplus H_K(M') = T\right) \leq \varepsilon \quad (\forall\ M \neq M', T)$

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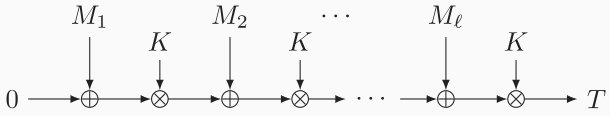
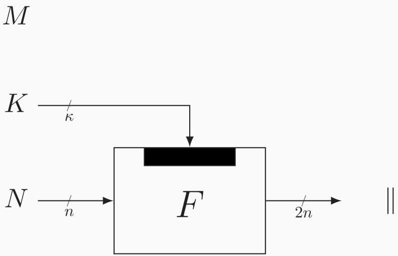
GHASH

- Addition and multiplication over finite field
- $\ell 2^{-t}$-XOR-universal [MV04]





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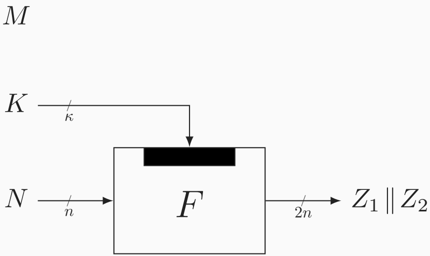

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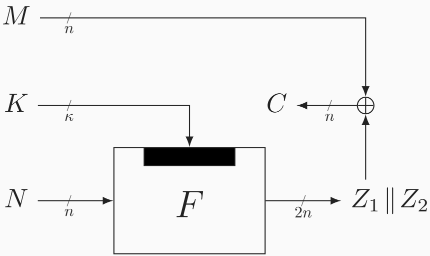

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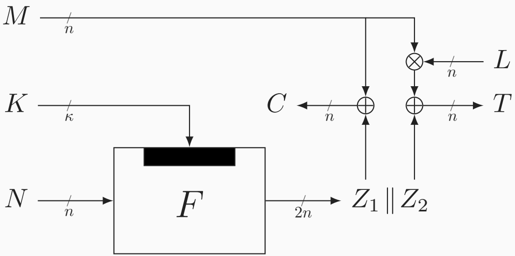

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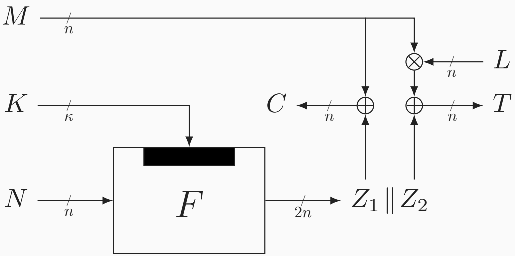

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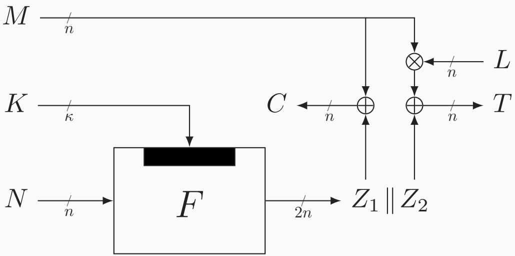

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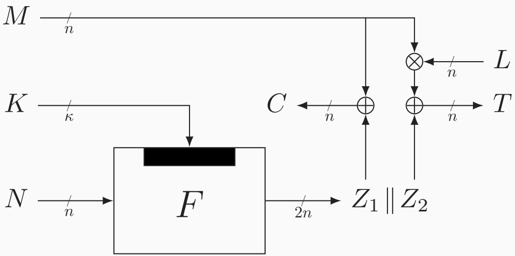

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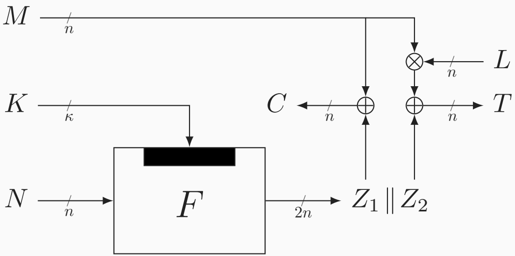

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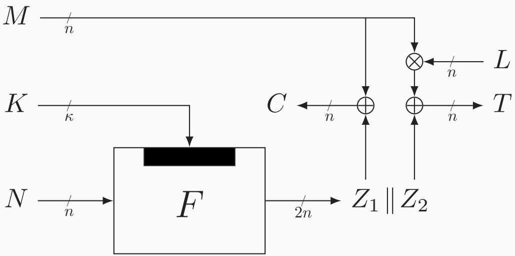

Confidentiality

- Consider new query (N, M)
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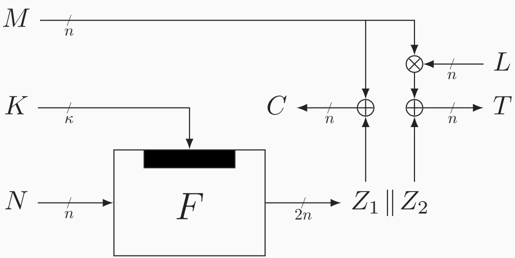

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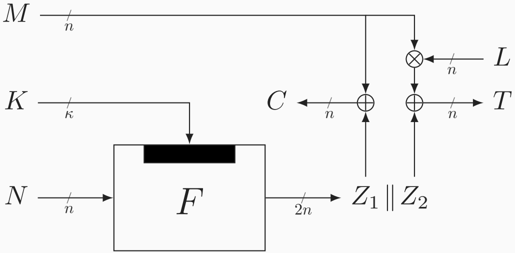

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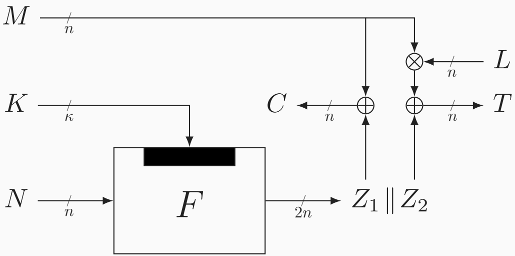

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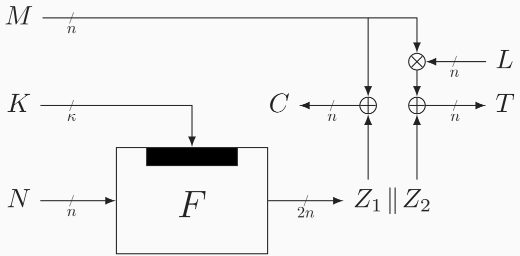

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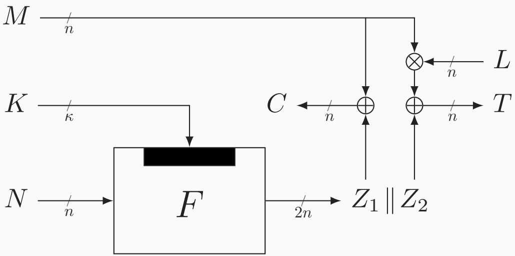

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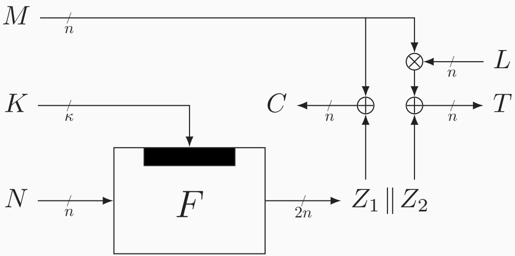

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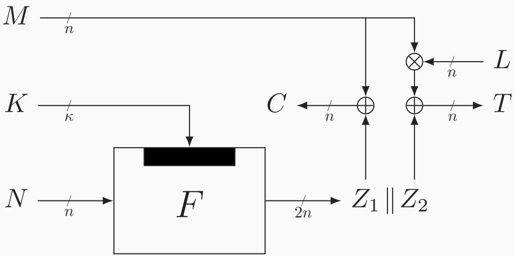

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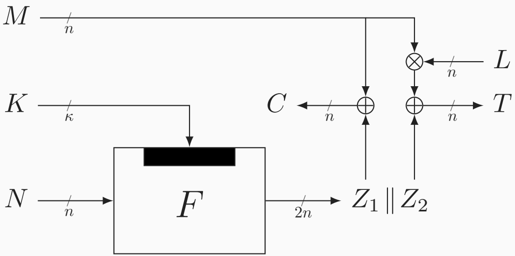

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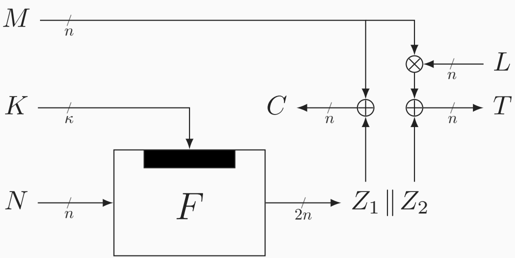

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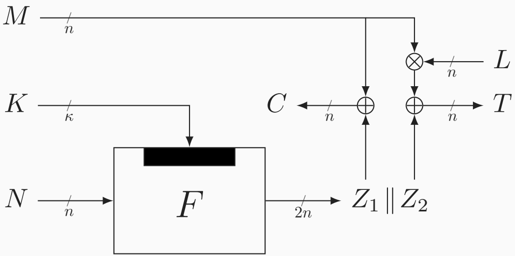

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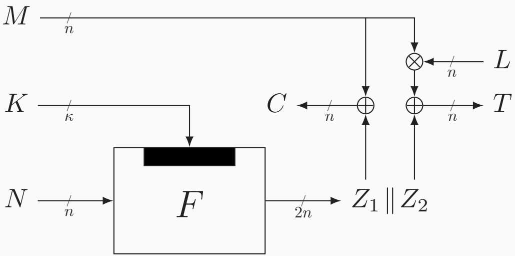
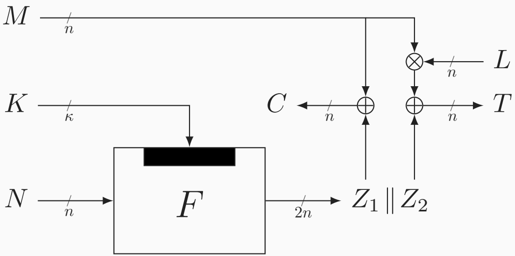


Suppose M is Variable-Length?

Simple Example: How to Support Variable Length?



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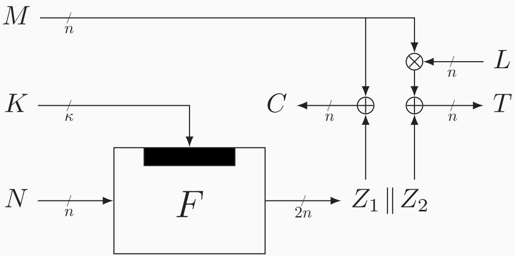

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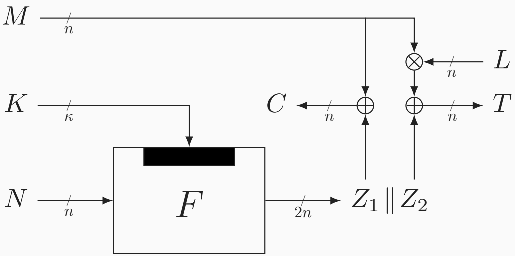

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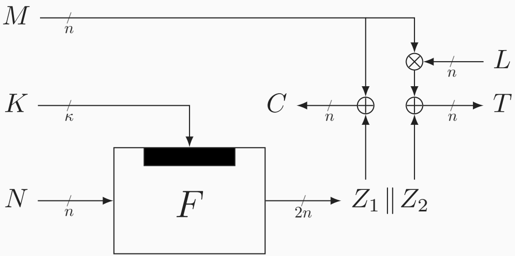

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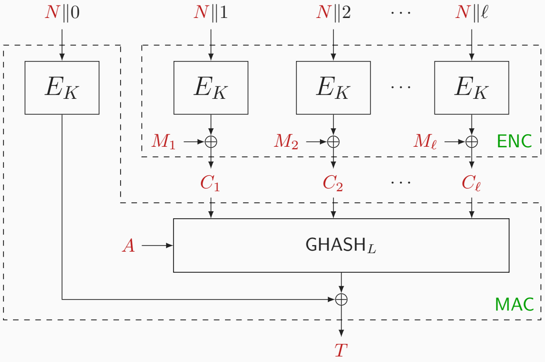
- Encrypt-*then*-MAC: $H_L(A, C)$
- Take CTR mode for F

GCM for 96-bit Nonce N



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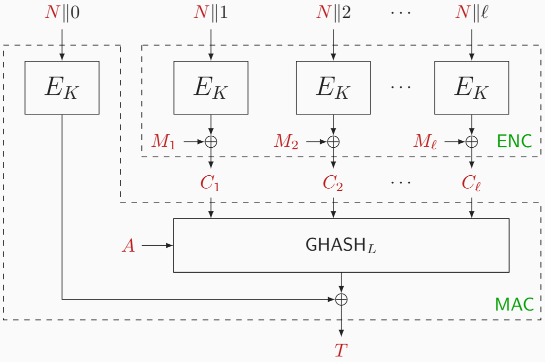

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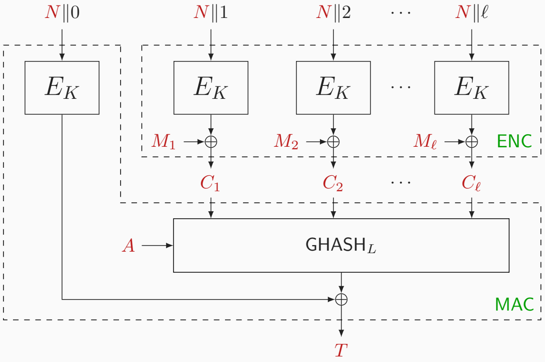

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- Note: equally popular is ChaCha20-Poly1305!

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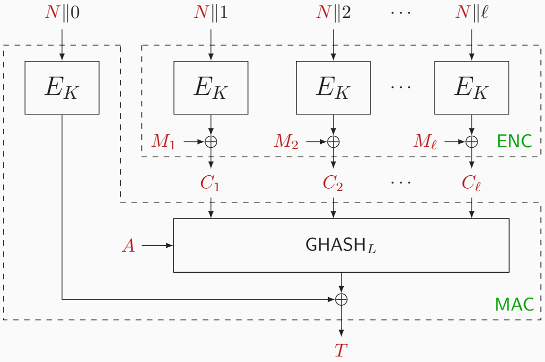

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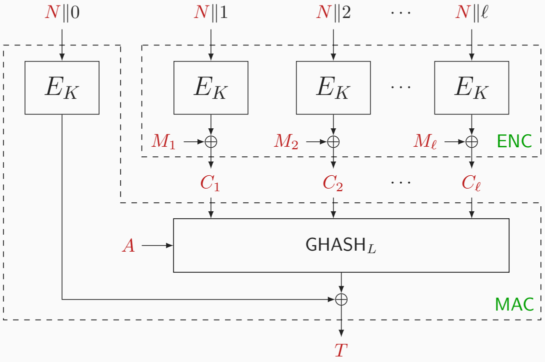

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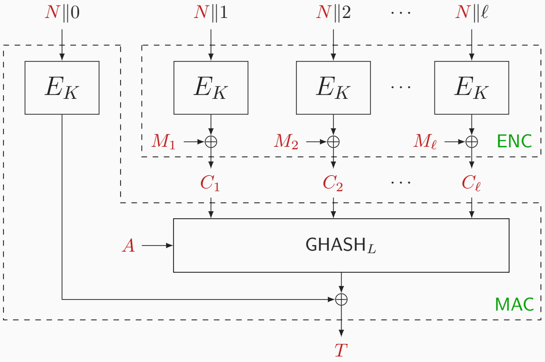

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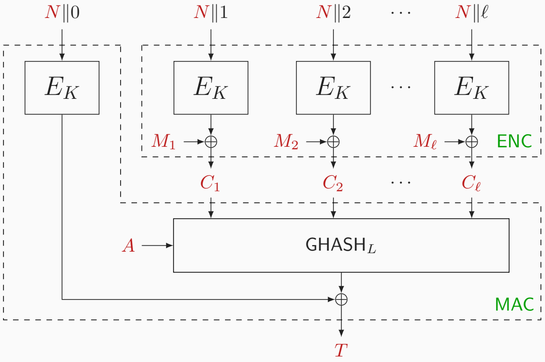

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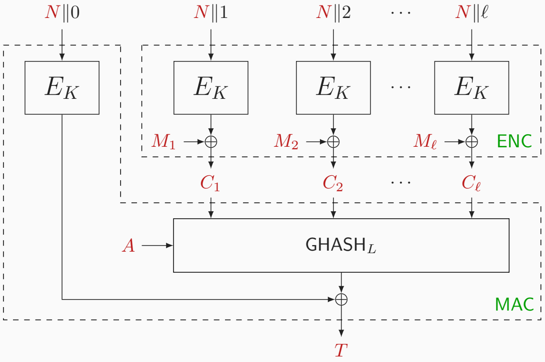

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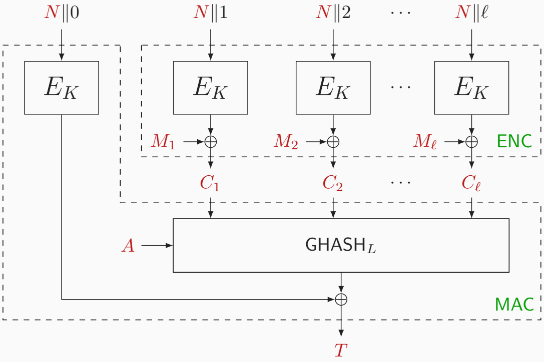

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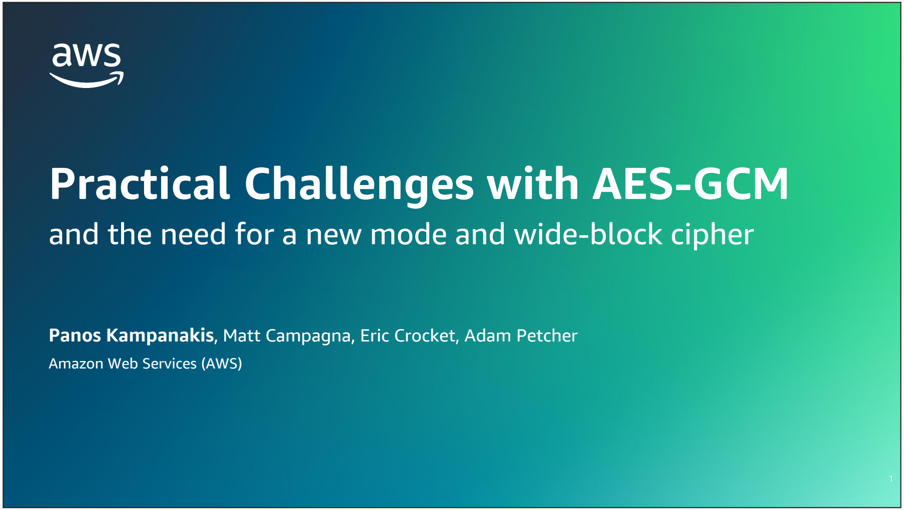
No Tag Truncation

- Easier subkey recovery [Fer05]
- Alternative hashing? [CMP23]

Use Case: Amazon Web Services [KCCP23]



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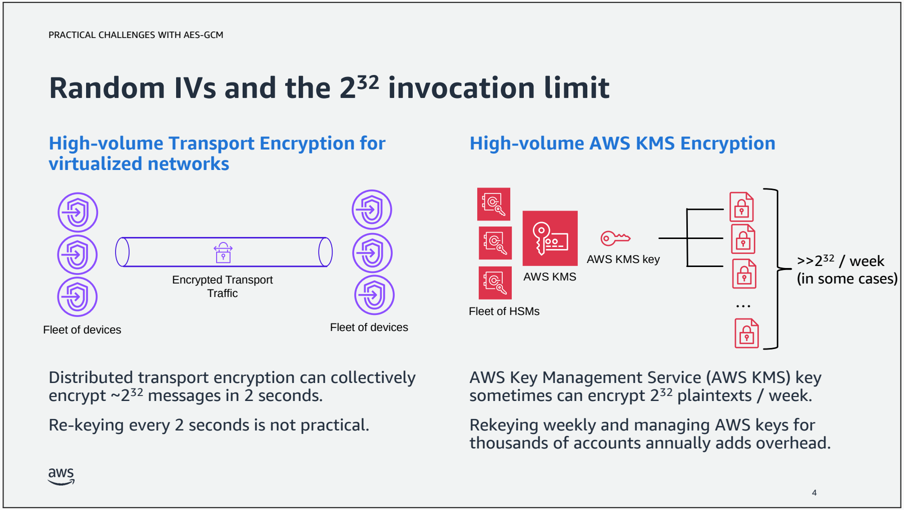

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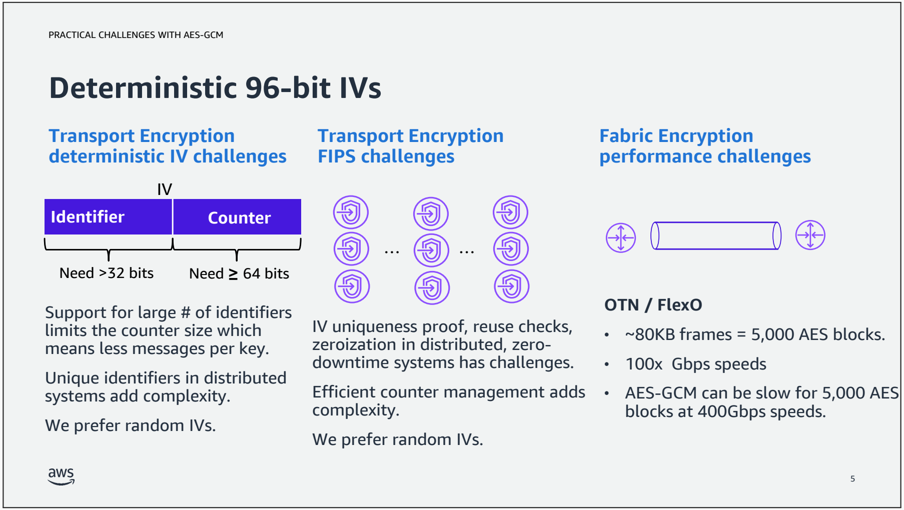

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Thank you for your attention!

Mihir Bellare and Björn Tackmann.
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Supporting Slides

H-Coefficient Technique and Security of Even-Mansour

H-Coefficient Technique

- Patarin [Pat91, Pat08b]
- Popularized by Chen and Steinberger [CS14]

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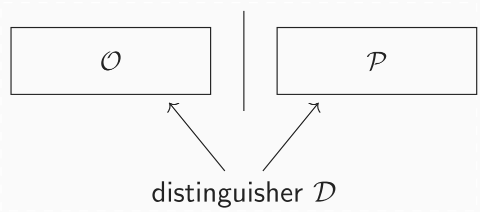

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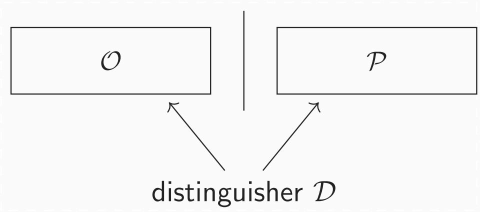

- Basic idea:
  - Each conversation defines a transcript τ

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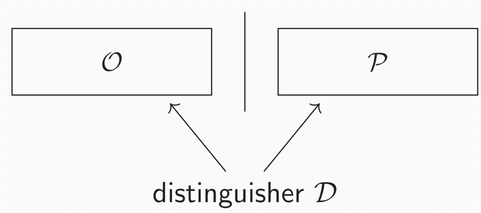

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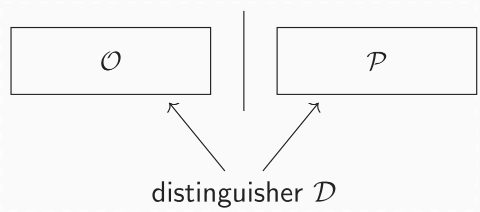

- Basic idea:
  - Each conversation defines a transcript τ
  - $\mathcal{O} \approx \mathcal{P}$ for most of the transcripts
  - Remaining transcripts occur with small probability

H-Coefficient Technique

- $\mathcal{D}$ is computationally unbounded and deterministic
- Complexity only measured by the number of queries
- Each conversation defines a transcript τ

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Lemma

Let $\varepsilon \geq 0$ be such that for all good transcripts τ:

$$\frac{\mathbf{Pr}\left(\mathcal{O} \text{ gives } \tau\right)}{\mathbf{Pr}\left(\mathcal{P} \text{ gives } \tau\right)} \geq 1 - \varepsilon$$

Then, $\Delta_{\mathcal{D}}(\mathcal{O}; P) \leq \varepsilon + \mathbf{Pr}\left(\text{bad transcript for } \mathcal{P}\right)$

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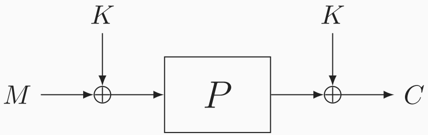
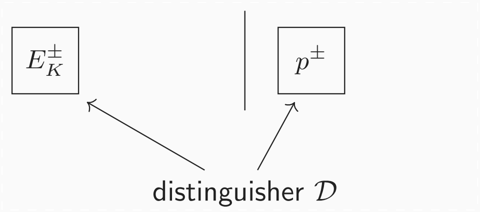
Trade-off: define bad transcripts smartly!

Security of Even-Mansour (1/6)

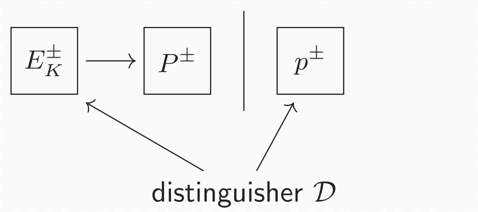
- Even-Mansour construction [EM91]:



$$E_K(M) = P(M \oplus K) \oplus K$$



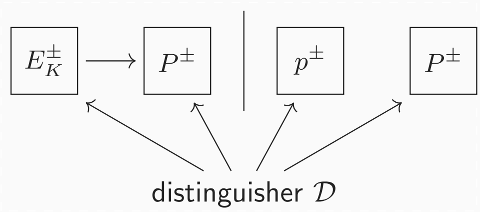
Slightly Different Security Model



Slightly Different Security Model

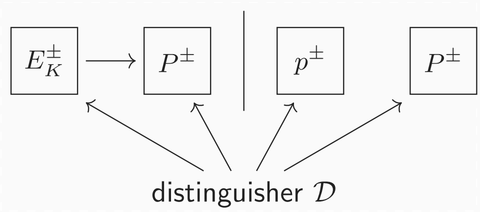
- Underlying permutation

Security of Even-Mansour (2/6)



Slightly Different Security Model

- Underlying permutation randomized
- Information-theoretic distinguisher $\mathcal{D}$
  - q construction queries
  - t offline evaluations $\approx t$ primitive queries



Slightly Different Security Model

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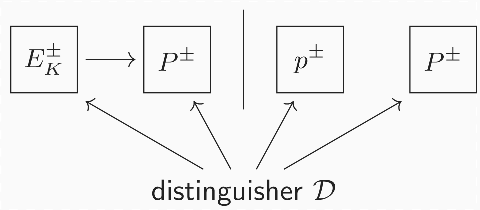
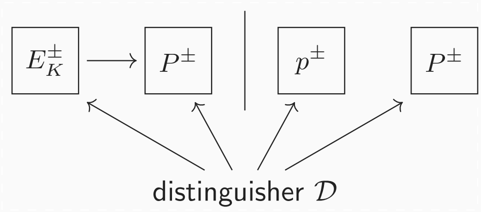
  - t offline evaluations $\approx t$ primitive queries
  - Unbounded computational power



- Two construction oracles: (E_K, E_K^{-1}) (for secret key K) and (p, p^{-1}) (secret)
- Two primitive oracles: (P, P^{-1}) (secret)

Security of Even-Mansour (3/6)



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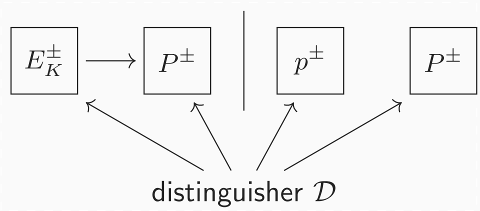
- $\mathcal{D}$ tries to determine which oracle it communicates with



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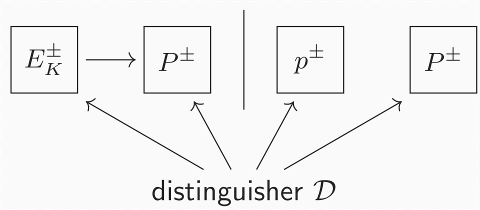
$$\mathbf{Adv}_E^{\mathrm{sprp}}(\mathcal{D}) = \Delta_{\mathcal{D}}\left(E_K, E_K^{-1}\ ;\ p, p^{-1}\right) = \left|\mathbf{Pr}\left(\mathcal{D}^{E_K, E_K^{-1}} = 1\right) - \mathbf{Pr}\left(\mathcal{D}^{p, p^{-1}} = 1\right)\right|$$



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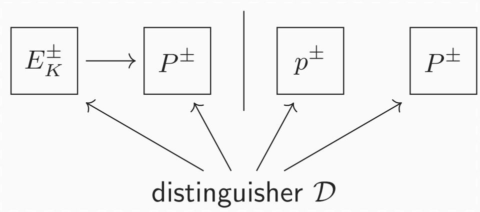
- $\mathbf{Adv}_E^{\mathrm{sprp}}(q, t)$: maximum advantage over any $\mathcal{A}$ with query/time complexity q/t



Theorem

For any distinguisher $\mathcal{D}$ making q queries to $E_K^\pm/p^\pm$ and t primitive queries

$$\mathbf{Adv}_E^{\mathrm{sprp}}(\mathcal{D}) = \Delta_{\mathcal{D}}(E_K^\pm, P^\pm; p^\pm, P^\pm) \leq ???$$

Security of Even-Mansour (5/6)

Step 1. Define how transcripts look like

Step 2. Define good and bad transcripts

Step 3. Upper bound $\mathbf{Pr}\,(\text{bad transcript for } (p^{\pm}, P^{\pm}))$

Step 4. Lower bound $\frac{\mathbf{Pr}\big((E_K^{\pm}, P^{\pm}) \text{ gives } \tau\big)}{\mathbf{Pr}\big((p^{\pm}, P^{\pm}) \text{ gives } \tau\big)} \geq 1 - \varepsilon$ ($\forall$ good τ)

1. **Define how transcripts look like**

- Construction queries:

$$\tau_E = \{(M_1, C_1), \ldots, (M_q, C_q)\}$$

- Primitive queries:

$$\tau_P = \{(X_1, Y_1), \ldots, (X_t, Y_t)\}$$

Security of Even-Mansour (6/6)

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  - After interaction of $\mathcal{D}$ with oracles: reveal the key
    - Real world $(E_K^{\pm}, P^{\pm})$: key used for encryption
    - Ideal world $(p^{\pm}, P^{\pm})$: dummy key $K \xleftarrow{\$} \{0,1\}^n$