Security of Encryption Modes and an Exposition of Proof Techniques

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Keyed Symmetric Cryptography

General Setting

- Two parties, Alice and Bob, communicate over a public channel
    - They have agreed on a joint key and use it to transmit data

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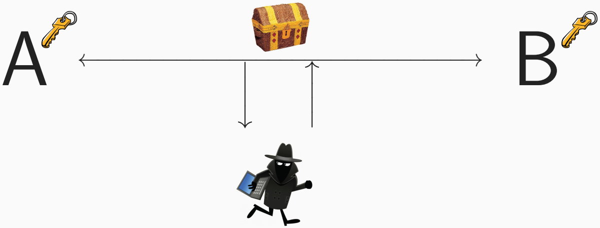

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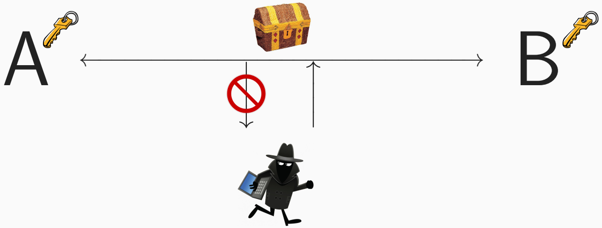

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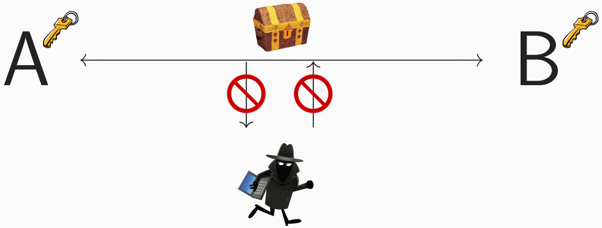

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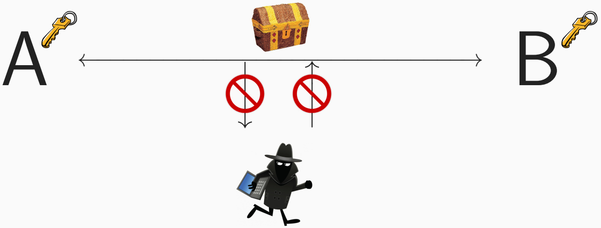

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In this presentation I will focus on confidentiality

One-Time Pad Encryption

Encryption:

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\begin{array}{rccccccccccccccc}
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Stream Ciphers

- Generate long keystream Z from short key K
- Much more practical!
- Security degrades:
  1. Key guessing still succeeds with probability $1/2^{|K|}$ but now with shorter key
  2. The stream cipher mechanism is another focal point of attack

Stream Cipher: Vigenère ($\approx$ 1553, Wikipedia)

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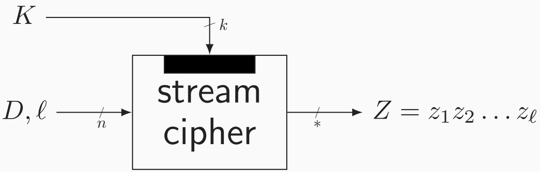
We need something more sophisticated!

How to Model Security?



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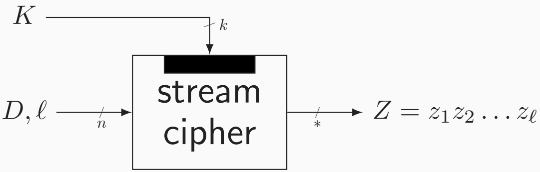

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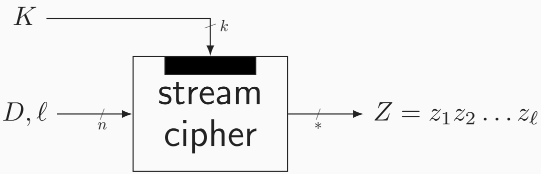

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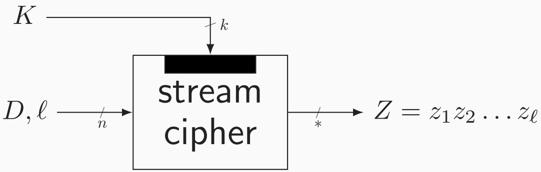

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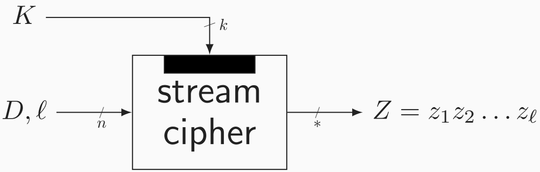
When is a stream cipher strong enough?

Stream Cipher Security, Intuition (1/3)



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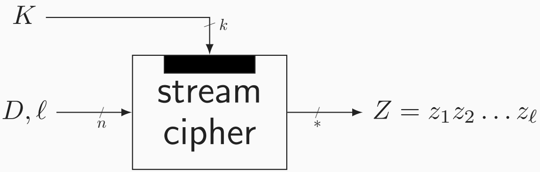

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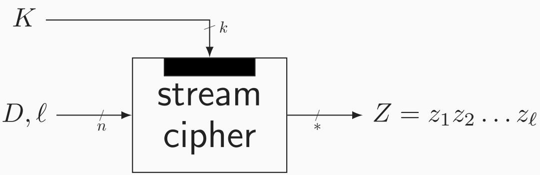

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- SC_K should behave like a random oracle

Random Oracle

- A database of input-output tuples
- Initially empty

D	Z
$\dots$	$\dots$
$\dots$	$\dots$
$\dots$	$\dots$
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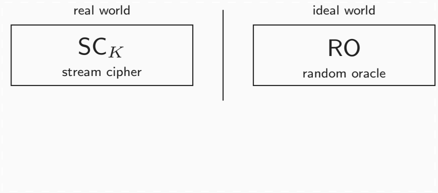
    - update (D, Z) in the list

Stream Cipher Security, Intuition (2/3)



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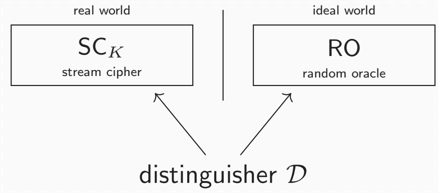

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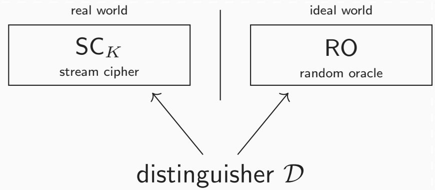

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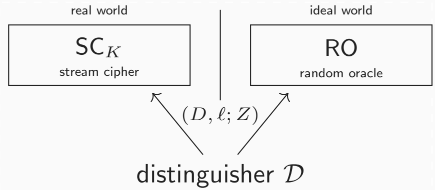

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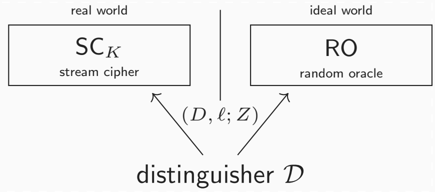

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  - At the end, $\mathcal{D}$ has to guess the outcome of the toss coin (head/tail)

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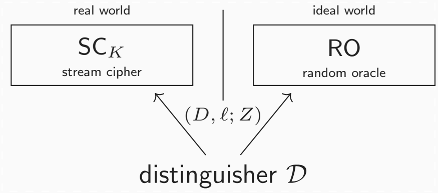

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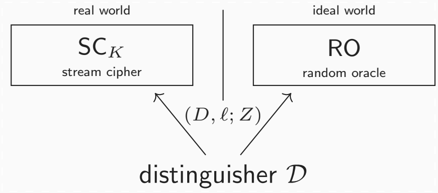

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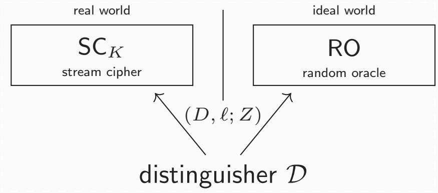

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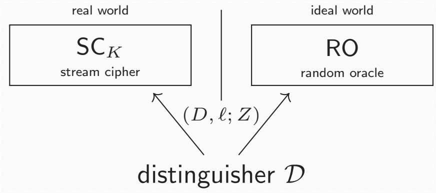

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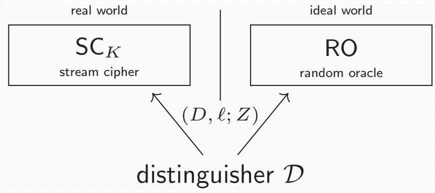

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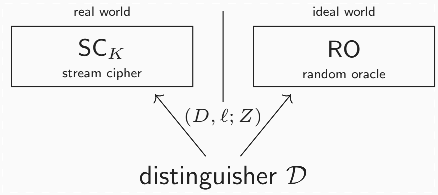
- $\mathcal{D}$ is limited by certain constraints
  - Data (or online) complexity q: total cost of queries $\mathcal{D}$ can make
  - Computation (or time) complexity t: everything that $\mathcal{D}$ can do "on its own"

Stream Cipher Security, Formal



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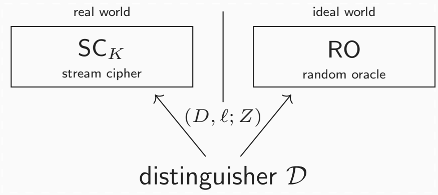

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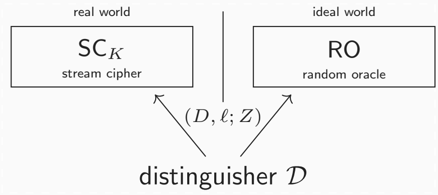

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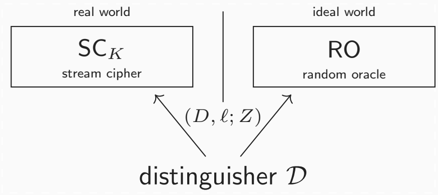
Stream Cipher Security, Formal



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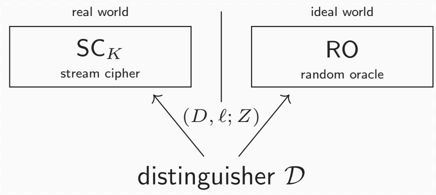

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- $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{SC}}(q, t)$: maximum advantage over any distinguisher with complexity q, t

Generic Stream Cipher Design

Generic Stream Cipher Design (1/2)

- Classical approach: LFSRs strengthened with non-linear component
- Modern approach: building construction from smaller cryptographic primitive

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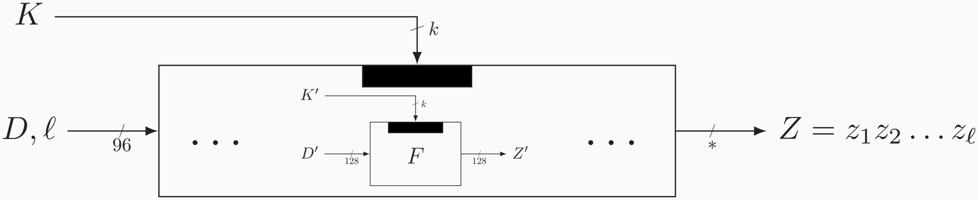
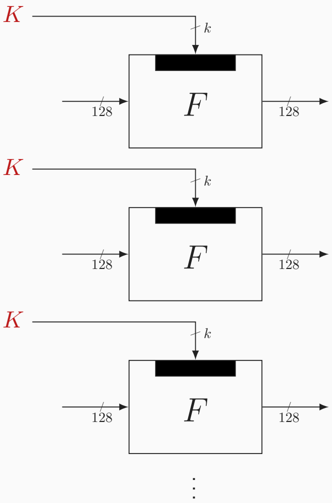
- Suppose (for the sake of argument):
  - we **know** how to build a strong stream cipher F with fixed-length output
  - we **want** to build a stream cipher with variable-length output



Design

- Feed K to primitive



Design

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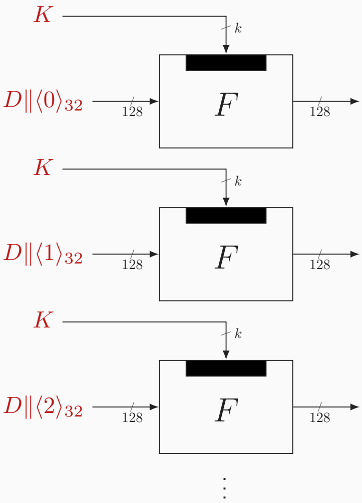

Design

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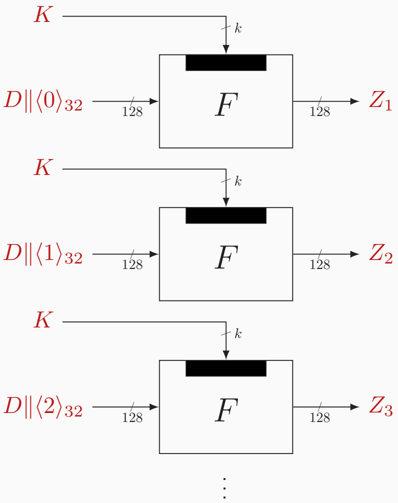
$$Z = Z_1 \parallel Z_2 \parallel Z_3 \parallel \cdots$$



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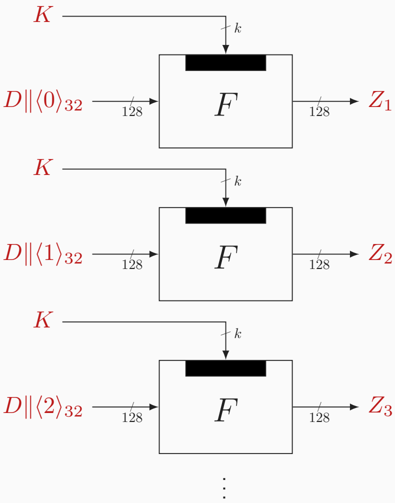

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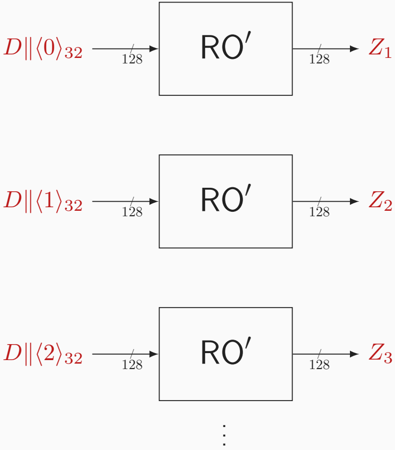

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Generic Stream Cipher Design (2/2)

Design

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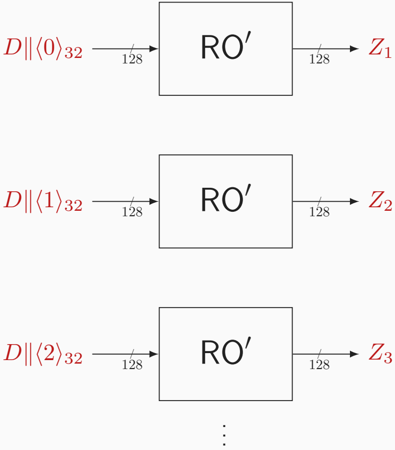

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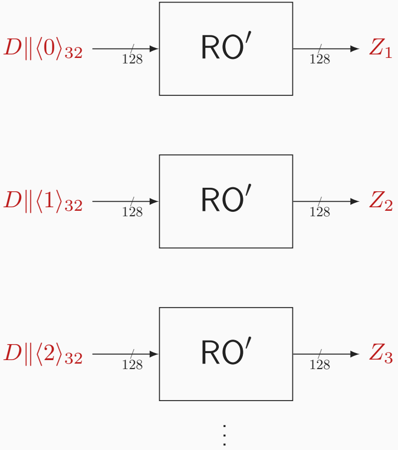

Security

- If F_K is hard to distinguish from a RO′
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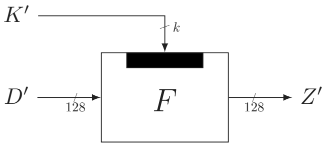
- For the purists: $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{SC}[F]}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(q,t')$

Design

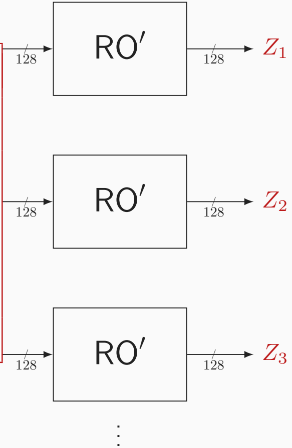
- Feed K to primitive
- Evaluate primitive as concatenated with co
- Concatenate outputs:

$Z = Z_1$

Unfortunately, we do not know how to easily construct a function



that behaves like a RO′



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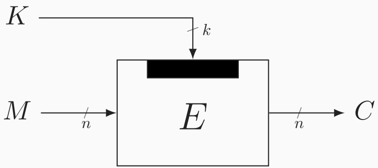

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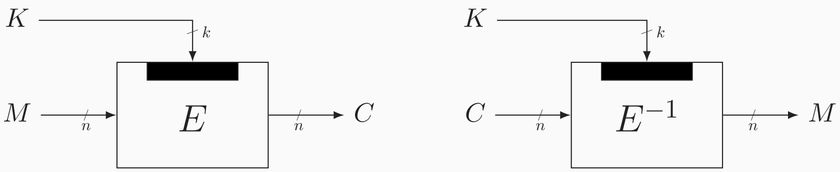

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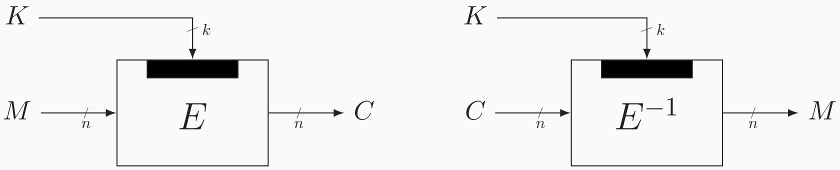

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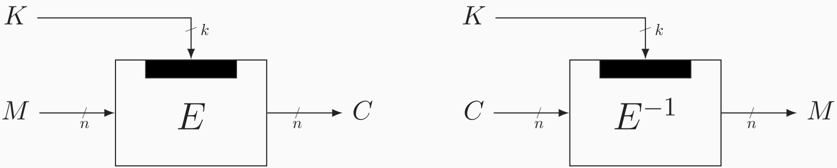

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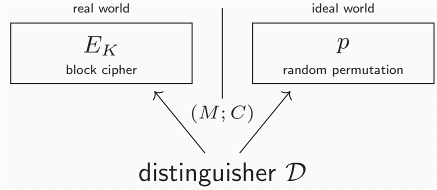
- A good block cipher should behave like a random permutation

Block Cipher Security



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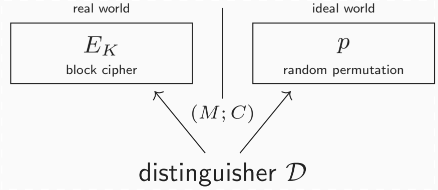

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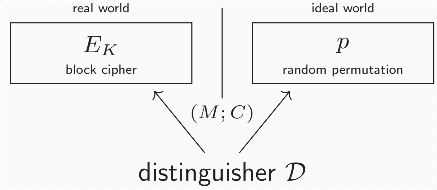

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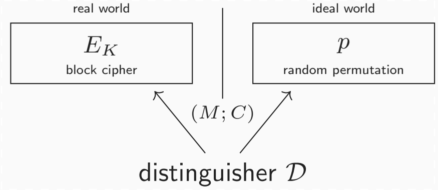

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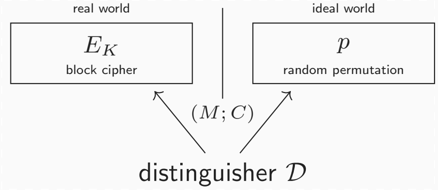

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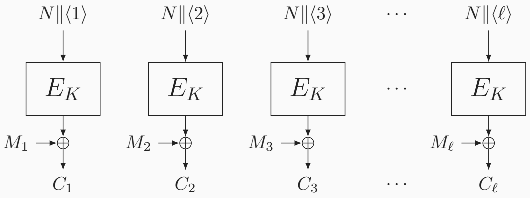
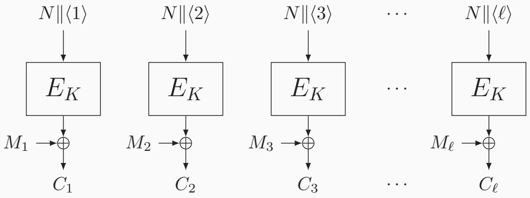
Counter Mode Encryption



Features

- Stream-based encryption mode
- Fully parallelizable (encryption and decryption) and extremely simple
- Decryption needs no E_K^{-1}

Counter (CTR) Mode



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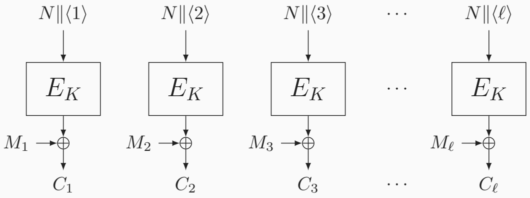

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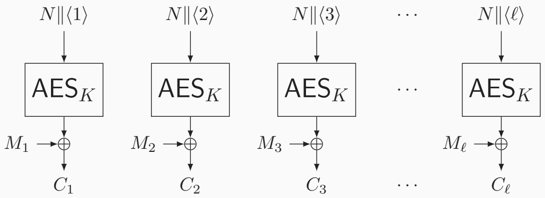
- Let us investigate that!

Security of Counter Mode Based on AES

- Let us consider counter mode based on AES: CTR[AES_K]



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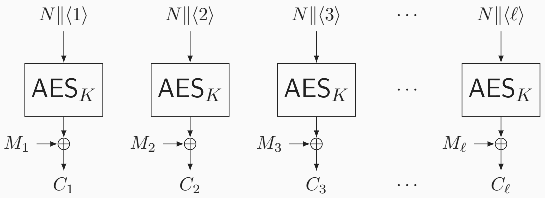

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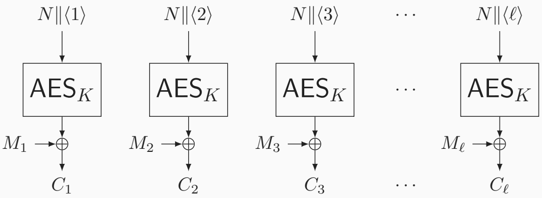
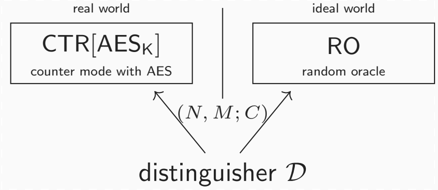


- We focus on the keystream generation portion
- Assumptions
  - Distinguisher never repeats nonce N
  - AES itself is sufficiently secure: $\mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q,t)$ is small



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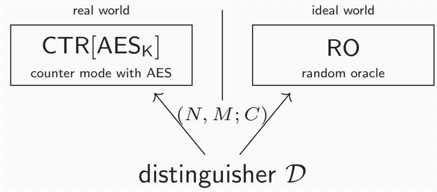
Security of Counter Mode Based on AES: Model



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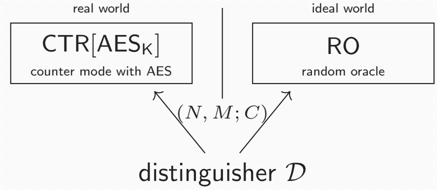

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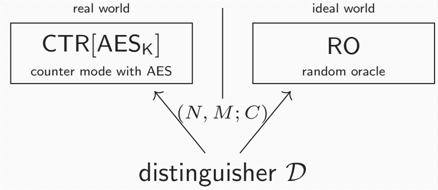

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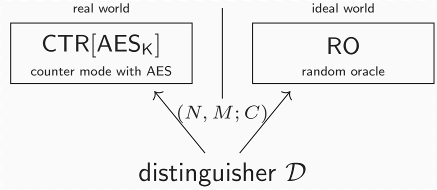

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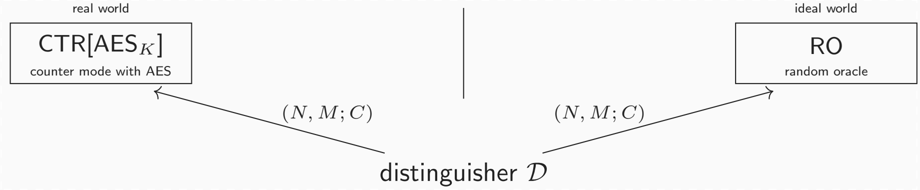
Proof: Overview



- For any (fixed) distinguisher $\mathcal{D}$ (later, we supremize over all), we have to bound:

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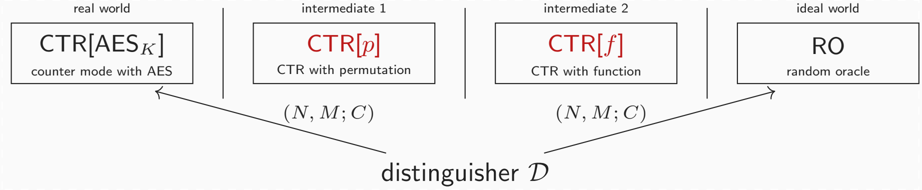

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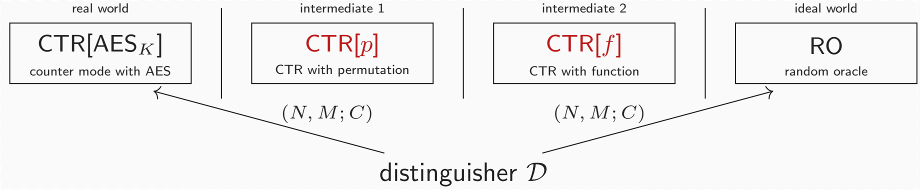

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- By the triangle inequality:

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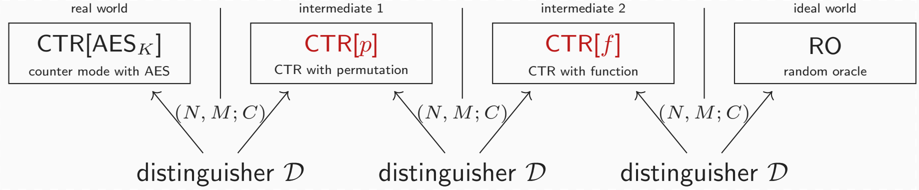

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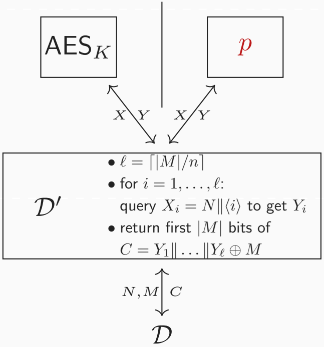
- $\mathcal{D}'$ simulates the oracles of $\mathcal{D}$:
- Once $\mathcal{D}$ makes its final guess, $\mathcal{D}'$ makes the same guess



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- $\mathcal{D}$'s goal: distinguish CTR[AES_K] from CTR[p]
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- $\mathcal{D}'$ simulates the oracles of $\mathcal{D}$:
- Once $\mathcal{D}$ makes its final guess, $\mathcal{D}'$ makes the same guess

- $\mathcal{D}'$ success probability turns out to be at least that of $\mathcal{D}$:

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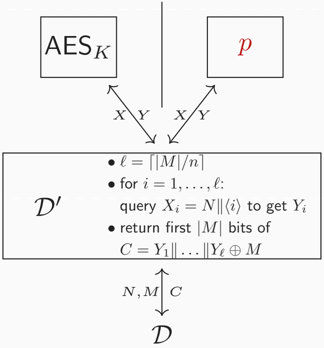

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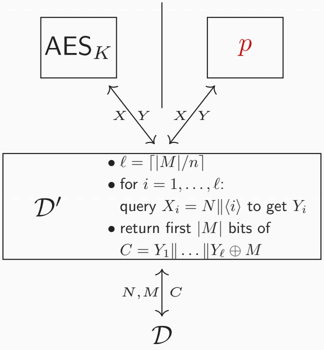
- But we have seen this distance before:

  $\Delta_{\mathcal{D}'}\left(\text{AES}_K\ ;\ p\right) = \mathbf{Adv}^{\text{prp}}_{\text{AES}}(\mathcal{D}') \leq \mathbf{Adv}^{\text{prp}}_{\text{AES}}(q, t')$

  (t' slightly larger than t)



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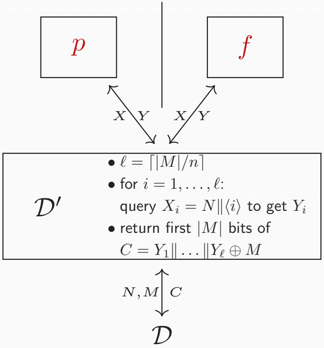

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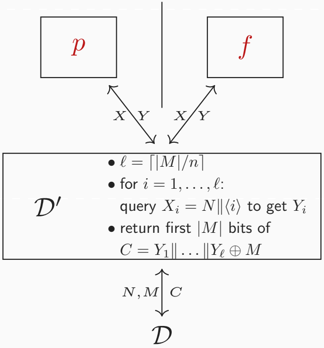

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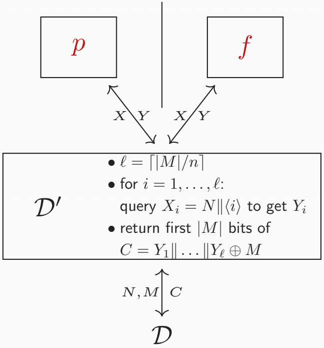
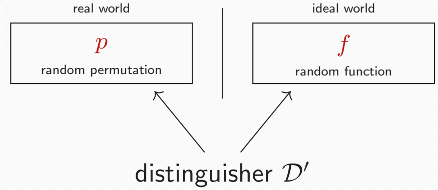
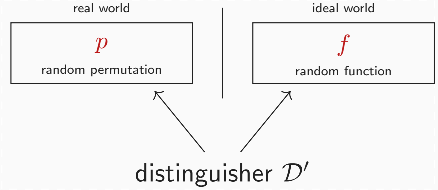
  $\Delta_{\mathcal{D}}(\text{CTR}[p]\ ;\ \text{CTR}[f]) \leq \Delta_{\mathcal{D}'}(p\ ;\ f)$
- This is a well-known distance, called the RP-RF switch





- Distinguisher $\mathcal{D}'$ gets q random n-bit samples:
  - real world: without replacement
  - ideal world: with replacement

Proof: From CTR$[p]$ to CTR$[f]$ (2/2)



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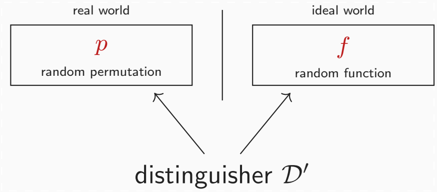
- The two worlds can only be distinguished if f ever outputs colliding samples



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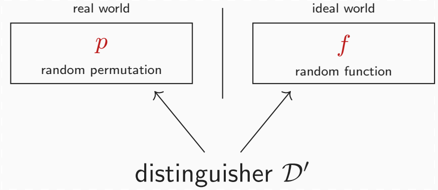

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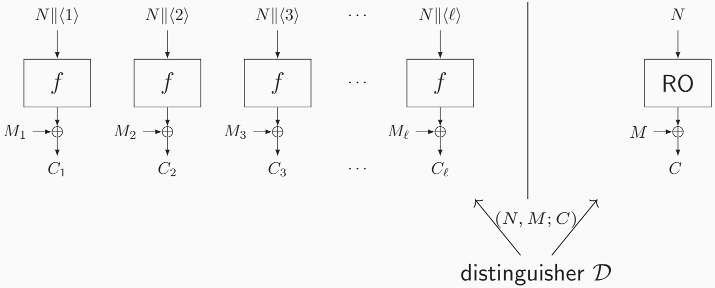
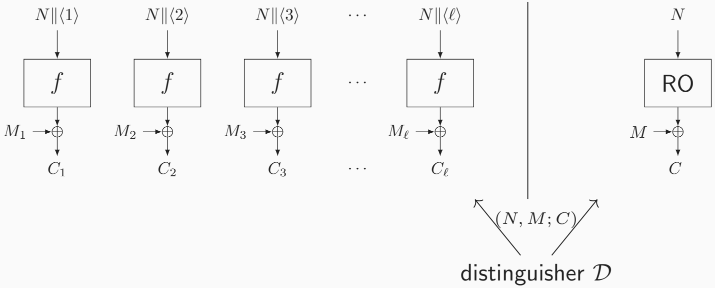


- In real world: f is a random function that is never evaluated for repeated $N\|\langle i\rangle$
- In ideal world: RO is a random oracle that is never evaluated for repeated N

Proof: From CTR$[f]$ to RO



- In real world: f is a random function that is never evaluated for repeated $N\|\langle i\rangle$
- In ideal world: RO is a random oracle that is never evaluated for repeated N
- Hence: $\Delta_{\mathcal{D}}\left(\text{CTR}[f]\ ;\ \text{RO}\right) = 0$

Proof: Conclusion

- Recall goal: bounding $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D})$ for any $\mathcal{D}$ querying q blocks in t time

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\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(\mathcal{D}) &= \Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\ ;\ \mathsf{RO}\right) \\
&\leq \Delta_{\mathcal{D}}\left(\mathsf{CTR[AES}_K\mathsf{]}\ ;\ \mathsf{CTR}[p]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[p]\ ;\ \mathsf{CTR}[f]\right) + \Delta_{\mathcal{D}}\left(\mathsf{CTR}[f]\ ;\ \mathsf{RO}\right) \\
&\leq \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q, t') + \binom{q}{2}/2^n + 0
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&\le \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q,t') + \binom{q}{2}/2^n + 0
\end{aligned}
$$

- As this reasoning holds for all distinguishers $\mathcal{D}$ querying q blocks in t time, we obtain:

$$
\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[AES]}}(q,t) \le \mathbf{Adv}^{\mathrm{prp}}_{\mathsf{AES}}(q,t') + \binom{q}{2}/2^n
$$

Beyond Birthday Bound Security

Birthday Paradox

For a random selection of 23 people, with a probability at least 50% two of them share the same birthday

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General Birthday Paradox

- Consider space $\mathcal{S} = \{0,1\}^n$
- Randomly draw q elements from $\mathcal{S}$
- Expected number of collisions:

$$\mathbf{Ex}\left[\text{collisions}\right] = \binom{q}{2}/2^n$$

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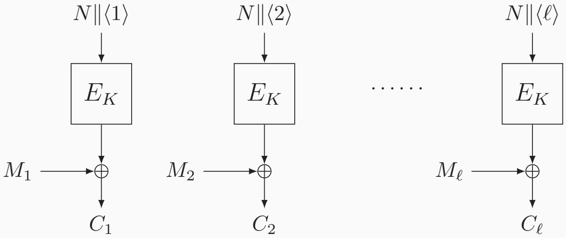
- Important phenomenon in cryptography

Counter Mode Based on Pseudorandom Permutation



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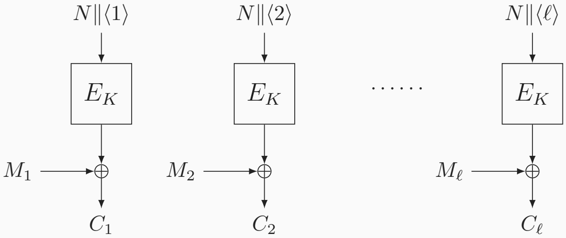

- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t) \le \mathbf{Adv}^{\mathrm{prp}}_{E}(q,t') + \binom{q}{2}/2^n$$

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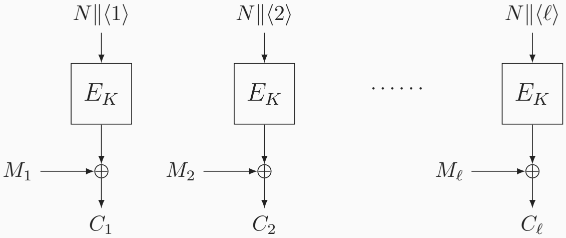

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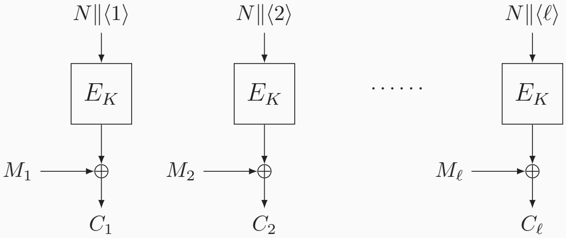
$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t) \le \mathbf{Adv}^{\mathrm{prp}}_{E}(q,t') + \binom{q}{2}/2^n$$

- $\mathsf{CTR}[E]$ is secure as long as:
  - E_K is a secure PRP
  - Number of encrypted blocks $q \ll 2^{n/2}$

Counter Mode Based on Pseudorandom Permutation



- $M_i \oplus C_i$ is distinct for all q blocks
- Unlikely to happen for random string

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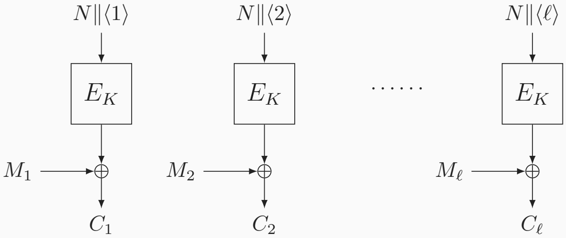
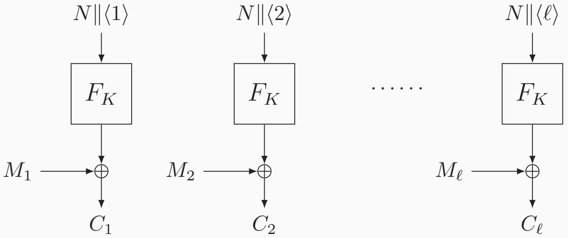


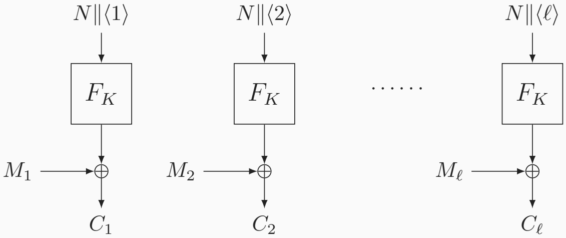
- $M_i \oplus C_i$ is distinct for all q blocks
- Unlikely to happen for random string
- Distinguishing attack in $q \approx 2^{n/2}$ blocks:

$$\binom{q}{2}/2^n \lesssim \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[E]}(q,t)$$

Counter Mode Based on Pseudorandom Function

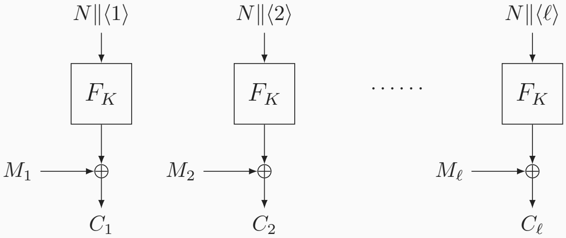


Counter Mode Based on Pseudorandom Function



- Security bound:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR}[F]}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(q,t')$$



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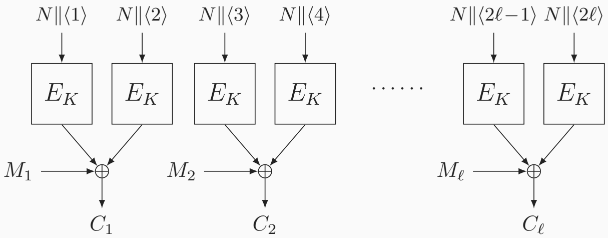
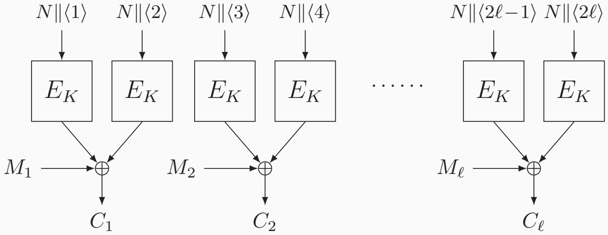
- CTR$[F]$ is secure as long as F_K is a secure PRF
- Birthday bound security loss disappeared

Counter Mode Based on XoP



- Security bound [Pat08a, DHT17]:

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CTR[XoP]}}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{\mathsf{XoP}}(q,t')$$



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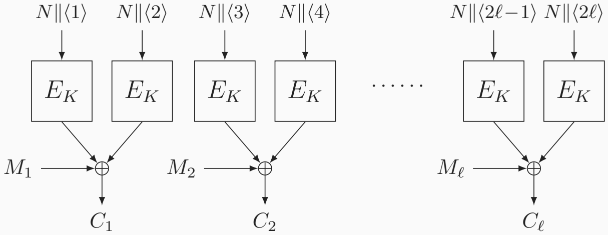

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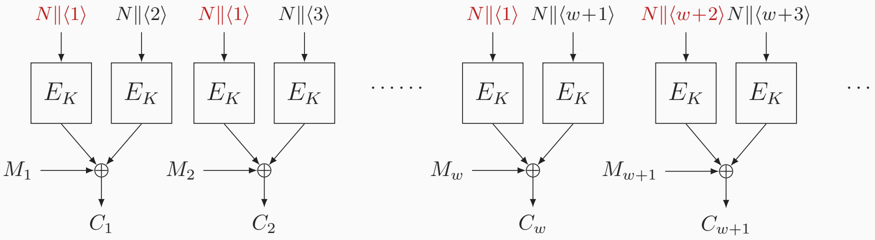
- Beyond birthday bound but 2x as expensive as CTR$[E]$

CENC by Iwata [Iwa06]



- One subkey used for $w \geq 1$ encryptions

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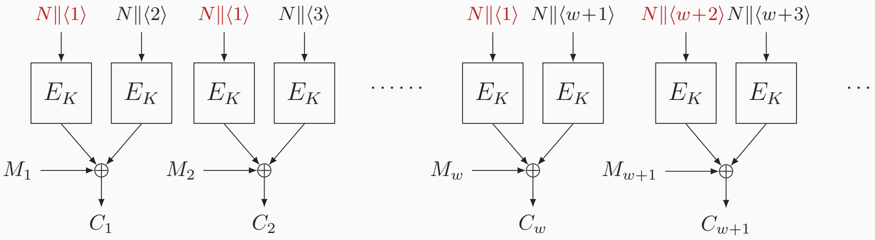

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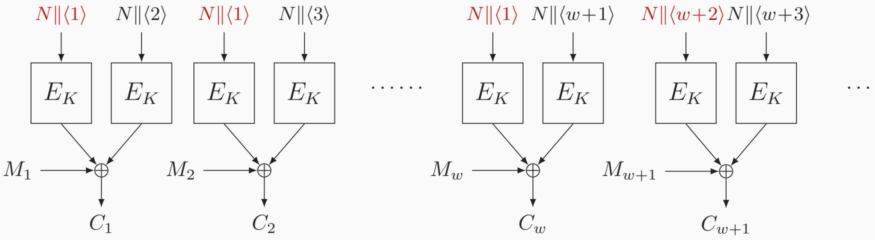

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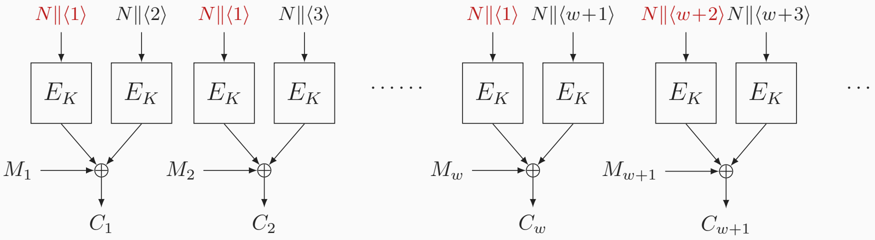

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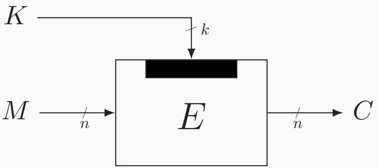
- Security of XoP and XoP$[w]$ can be proven using mirror theory [Pat03]

Accordion Modes

Block Ciphers



- Message M encrypted to ciphertext C with secret key K
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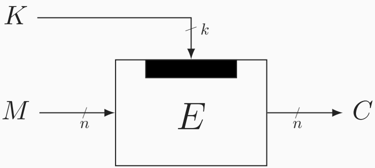
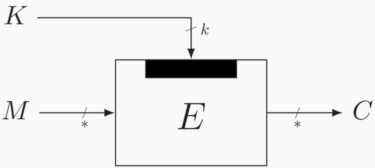
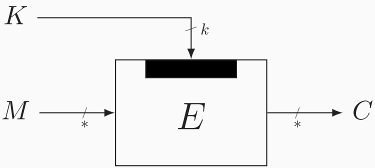


- Message M encrypted to ciphertext C with secret key K
- Fixed block size
- In order to encrypt variable sized messages, we need a mode of operation
  - These modes require a nonce

Wide Block Ciphers

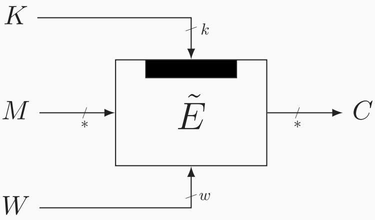


- Alternatively, we can design a wide block cipher
- A wide block cipher is a block cipher with a variable block size



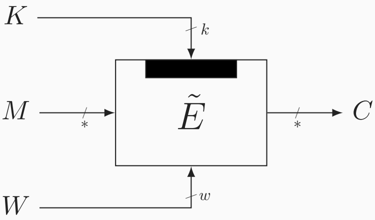
- Alternatively, we can design a wide block cipher
- A wide block cipher is a block cipher with a variable block size
- Every part of the output (ideally) depends on every part of the input

Tweakable Wide Block Ciphers



- A tweakable wide block cipher additionally has a tweak
- Tweak W public, ciphertext completely changes with a different tweak

Tweakable Wide Block Ciphers



- A tweakable wide block cipher additionally has a tweak
- Tweak W public, ciphertext completely changes with a different tweak
- Useful for e.g. disk encryption, where every sector gets its own tweak

NIST's Incentive to Develop Accordion Mode

- March 2024: NIST announced quest for tweakable wide block ciphers
- There is a workshop right now aimed to discuss ideas on requirements, designs, security goals, targets, ...

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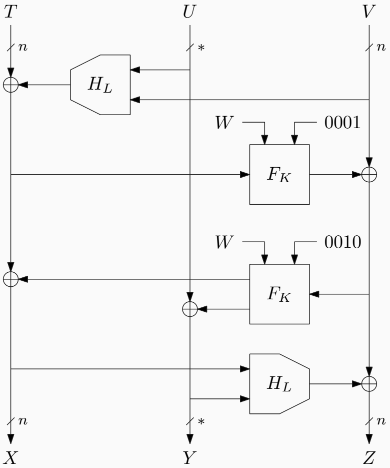
Now: high-level idea of our recent proposals

Docked Double Decker [GDM19]



Building Blocks

- F_K: stream cipher
- H_L: universal hash

Construction

- Feistel-like structure
- Outer lanes of fixed size
- Inner lane of variable size

Goals and Hurdles

Goals

- Instantiation using components as used in NIST standardized schemes:
  - AES [DR02, DR20]
  - Operations in binary extension fields, e.g., as in GHASH [MV04]

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- Instantiation using components as used in NIST standardized schemes:
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- Present birthday bound secure $ddd\text{-}AES$ and beyond birthday bound secure $bbb\text{-}ddd\text{-}AES$ that seamlessly fit NIST's accordion idea

Goals and Hurdles

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  - AES [DR02, DR20]
  - Operations in binary extension fields, e.g., as in GHASH [MV04]
- Present birthday bound secure $ddd\text{-}AES$ and beyond birthday bound secure $bbb\text{-}ddd\text{-}AES$ that seamlessly fit NIST's accordion idea

Hurdles

- AES is not a tweakable blockcipher
- AES is rather small (circular reasoning?)
- AES in typical stream cipher modes only gives birthday bound security

Efficient Instantiations of Docked Double Decker [DMMT24]

ddd-AES

- H_L instantiated using Polyval (as in GCM-SIV)
- F_K instantiated as variant of CTR: tweak used to randomize inputs to AES_K

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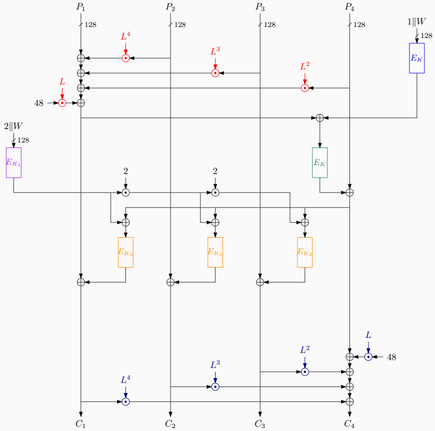
Instantiations turn out to be very competitive and well parallelizable

Implementation Design of *ddd-AES* (512-Bit Message)



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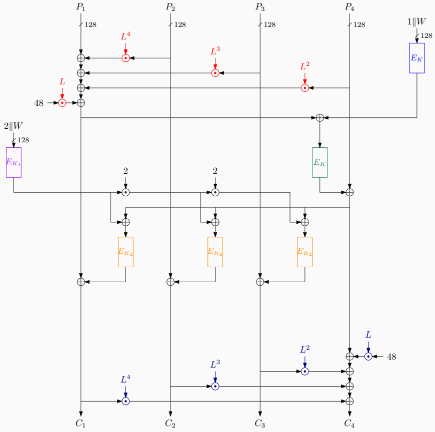
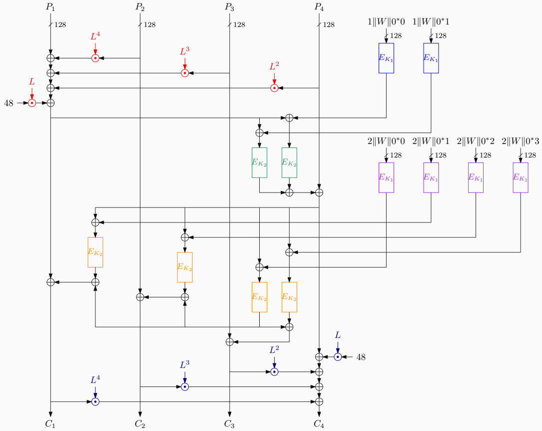
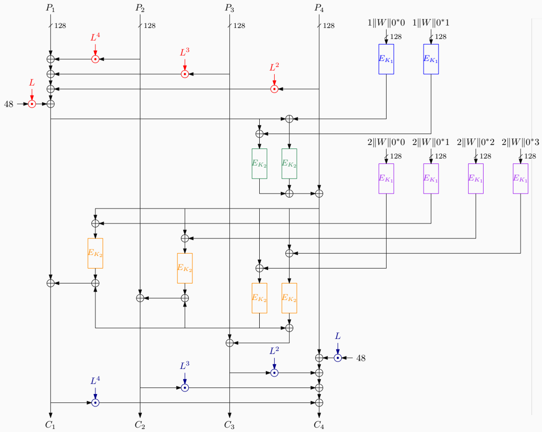


Implementation Design of $bbb\text{-}ddd\text{-}AES$ (512-Bit Message)



Implementation Design of $bbb\text{-}ddd\text{-}AES$ (512-Bit Message)



Conclusion

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Provable Security in Symmetric Cryptography

- Basic modes proved secure using quite simple ideas
- More sophisticated modes require nice tricks in graph theory
- Often this boils down to trying to upper or lower bound solutions

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- Accordion modes
- Arithmetization-oriented modes

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Thank you for your attention!

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Mirror Theory (Intuition)

System of Equations

- Consider r distinct unknowns $\mathcal{P} = \{P_1, \ldots, P_r\}$
- Consider a system of q equations of the form:

$$
\begin{aligned}
P_{a_1} \oplus P_{b_1} &= \lambda_1 \\
P_{a_2} \oplus P_{b_2} &= \lambda_2 \\
&\vdots \\
P_{a_q} \oplus P_{b_q} &= \lambda_q
\end{aligned}
$$

for some surjection $\varphi : \{a_1, b_1, \ldots, a_q, b_q\} \to \{1, \ldots, r\}$

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Goal

- Lower bound on the number of solutions to $\mathcal{P}$

Mirror Theory

- Extremely powerful lower bound

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- First introduced by Patarin in 2003 [Pat03]

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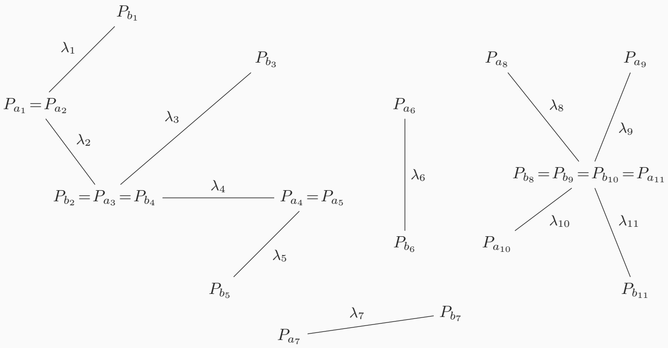
Now: graph-based intuition behind mirror theory

System of Equations

- r distinct unknowns $\mathcal{P} = \{P_1, \ldots, P_r\}$
- System of equations $P_{a_i} \oplus P_{b_i} = \lambda_i$
- Surjection $\varphi : \{a_1, b_1, \ldots, a_q, b_q\} \to \{1, \ldots, r\}$

Graph Based View



Mirror Theory: Toy Example 1

- System of equations:

$$P_a \oplus P_b = \lambda_1$$
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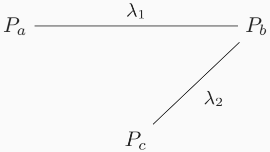

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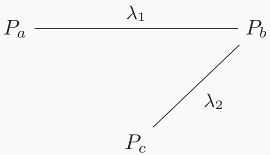

If $\lambda_1 = 0$ or $\lambda_2 = 0$ or $\lambda_1 = \lambda_2$

- Contradiction: $P_a = P_b$ or $P_b = P_c$ or $P_a = P_c$
- Scheme is degenerate

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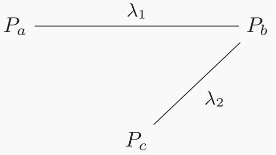

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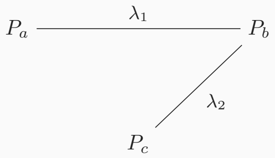
If $\lambda_1, \lambda_2 \neq 0$ and $\lambda_1 \neq \lambda_2$

- 2^n choices for P_a

- System of equations:

$$P_a \oplus P_b = \lambda_1$$
$$P_b \oplus P_c = \lambda_2$$



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- Fixes $P_b = \lambda_1 \oplus P_a$ (which is $\neq P_a$ as desired)

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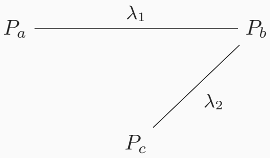

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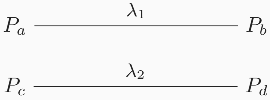
If $\lambda_1, \lambda_2 \neq 0$ and $\lambda_1 \neq \lambda_2$

- 2^n choices for P_a
- Fixes $P_b = \lambda_1 \oplus P_a$ (which is $\neq P_a$ as desired)
- Fixes $P_c = \lambda_2 \oplus P_b$ (which is $\neq P_a, P_b$ as desired)

Mirror Theory: Toy Example 2

- System of equations:

$$P_a \oplus P_b = \lambda_1$$
$$P_c \oplus P_d = \lambda_2$$



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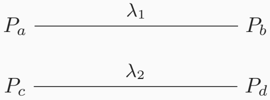

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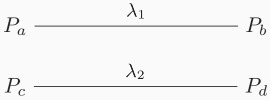

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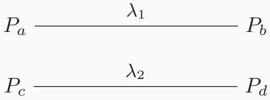

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- 2^n choices for P_a (which fixes P_b)
- For P_c and P_d we require
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  - $P_d = \lambda_2 \oplus P_c \neq P_a, P_b$

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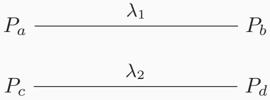


If $\lambda_1 = 0$ or $\lambda_2 = 0$

- Contradiction: $P_a = P_b$ or $P_b = P_c$
- Scheme is degenerate

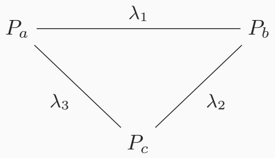
If $\lambda_1, \lambda_2 \neq 0$

- 2^n choices for P_a (which fixes P_b)
- For P_c and P_d we require
  - $P_c \neq P_a, P_b$
  - $P_d = \lambda_2 \oplus P_c \neq P_a, P_b$
- At least $2^n - 4$ choices for P_c (which fixes P_d)

- System of equations:

$$P_a \oplus P_b = \lambda_1$$
$$P_b \oplus P_c = \lambda_2$$
$$P_c \oplus P_a = \lambda_3$$

- Assume $\lambda_i \neq 0$ and $\lambda_i \neq \lambda_j$



Mirror Theory: Toy Example 3

- System of equations:

$$P_a \oplus P_b = \lambda_1$$
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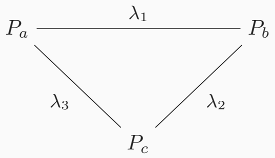

If $\boldsymbol{\lambda_1 \oplus \lambda_2 \oplus \lambda_3 \neq 0}$

- Contradiction: equations sum to $0 = \lambda_1 \oplus \lambda_2 \oplus \lambda_3$
- Scheme contains a circle

Mirror Theory: Toy Example 3

- System of equations:

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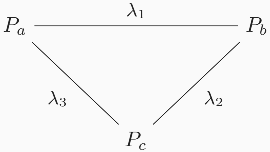

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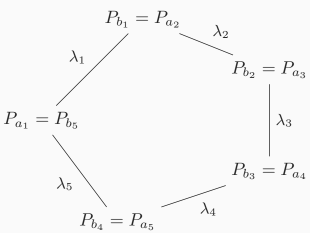
If $\boldsymbol{\lambda_1 \oplus \lambda_2 \oplus \lambda_3 = 0}$

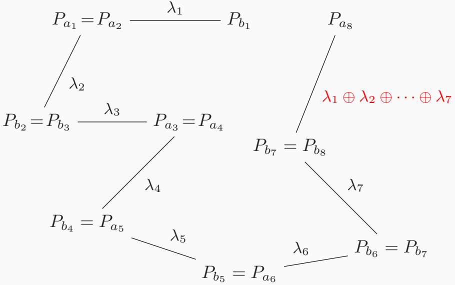
- One redundant equation, no contradiction
- Removing this equation brings us back at toy example 1

Mirror Theory: Two Problematic Cases

Circle



Degeneracy



System of Equations

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- Surjection $\varphi : \{a_1, b_1, \ldots, a_q, b_q\} \to \{1, \ldots, r\}$

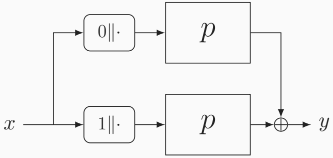
Main Result [CDN+23]

If the system of equations is circle-free and non-degenerate, the number of solutions to $\mathcal{P}$ is at least

$$\frac{(2^n)_r}{2^{nq}}$$

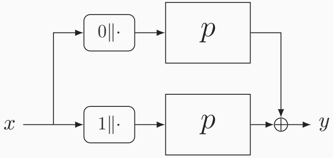
provided the maximum tree size ξ satisfies $\xi^2 \lesssim \min\{2^n/(12r), 2^{n/2}/n\}$

Mirror Theory Applied to XoP



General Setting

- Distinguisher gets transcript $\tau = \{(x_1, y_1), \ldots, (x_q, y_q)\}$



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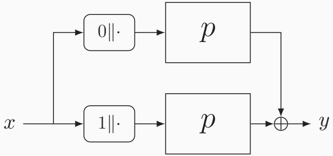
- Each tuple relates to $0\|x_i \mapsto p(0\|x_i) =: P_{a_i}$ and $1\|x_i \mapsto p(1\|x_i) =: P_{b_i}$

Mirror Theory Applied to XoP



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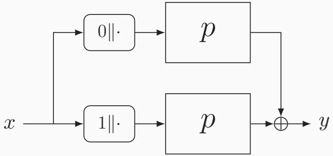

General Setting

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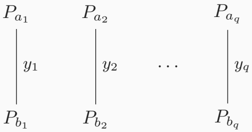
- System of q equations $P_{a_i} \oplus P_{b_i} = y_i$
- Inputs to p are all distinct: $2q$ unknowns

Mirror Theory Applied to XoP



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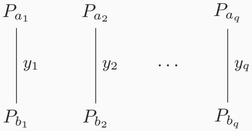
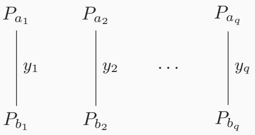


Applying Mirror Theory

- Circle-free: no collisions in inputs to p
- Non-degenerate: provided that $y_i \neq 0$ $(\forall i)$

  $\longrightarrow$ Call this a bad transcript
- Maximum tree size 2

Mirror Theory Applied to XoP



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- Circle-free: no collisions in inputs to p
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  $\longrightarrow$ Call this a bad transcript
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Mirror Theory Applied to XoP

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Let $\varepsilon \geq 0$ be such that for all good transcripts τ:

$$\frac{\mathbf{Pr}\left(\text{XoP gives } \tau\right)}{\mathbf{Pr}\left(f \text{ gives } \tau\right)} \geq 1 - \varepsilon$$

Then, $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{XoP}}(q) \leq \varepsilon + \mathbf{Pr}\left(\text{bad transcript for } f\right)$

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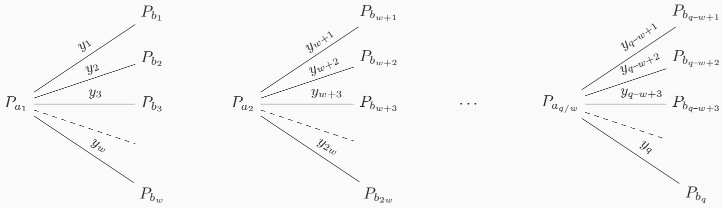
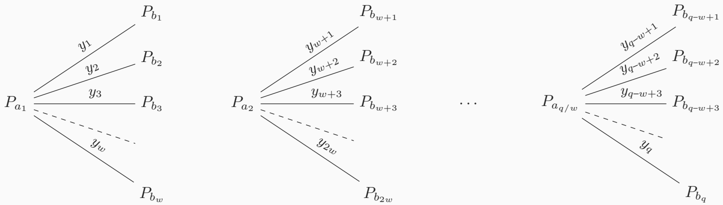
Mirror Theory Applied to CENC





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- Circle-free: no collisions in inputs to p
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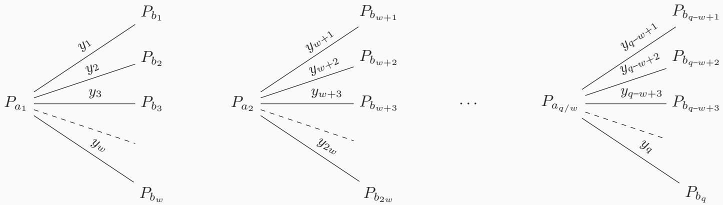

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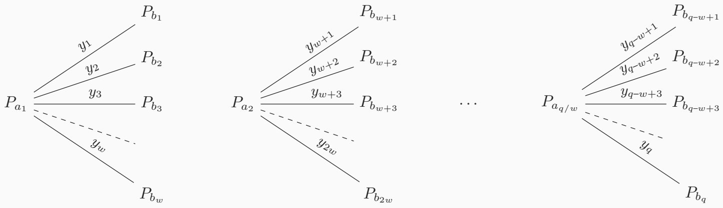

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- H-coefficient technique: $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CENC}}(q) \leq q/2^n + wq/2^{n+1}$

Accordion Modes (Instantiations)

Universal Hash Function Instantiation for Both $ddd\text{-}AES$ and $bbb\text{-}ddd\text{-}AES$

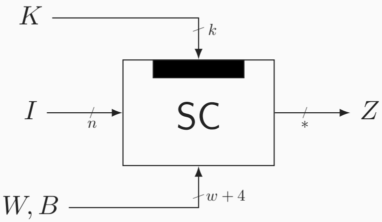
Polyval [GLL17]

- Operates on finite field $GF(2^{128})[x]/(x^{128} + x^{127} + x^{126} + x^{121} + 1)$
- Defined as follows, for a padded message $(I_1, I_2, \ldots, I_s)$:

$$\text{Polyval}_L(I_1, I_2, \ldots, I_s) = \sum_{i=1}^{s} \left(L^{s-i+1} \cdot I_i \cdot x^{-128\cdot(s-i+1)} \right)$$

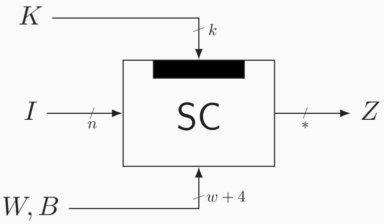
- We use zero-padding with length encoding

Recall Goal



Stream Cipher Instantiation

Recall Goal



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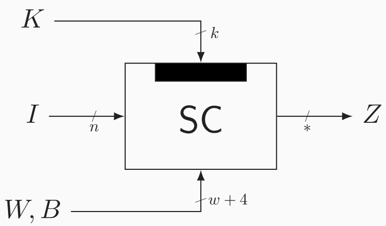
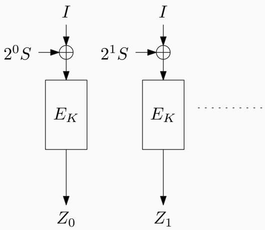


- Construction should be built on top of AES
- We give one construction with birthday bound security
  one construction with beyond birthday bound security

XE-style [Rog04] Tweakable Blockcipher in Counter Mode

- Let $S = E_K(B\|W)$



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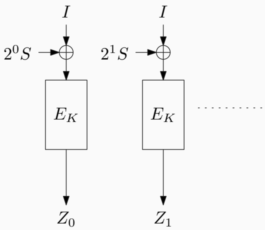

- Stream cipher (and thus ddd-AES) is $2^{n/2}$ PRF-secure

Bonus: Extension $ddd\text{-}AES^+$ to Accommodate Variable-Length Tweaks

- ddd-AES almost seamlessly fits NIST's accordion idea
- Only thing missing: variable-length tweaks

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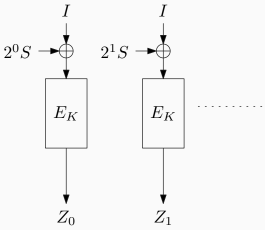
XE$^+$-style [Rog04] Tweakable Blockcipher in Counter Mode

- Pad B, W into $(W_0, W_1, \ldots, W_{l-1} \| B' \| 0^*)$ with $B' = B \oplus 1000$
- Let $S = E_K(W_0\|0) \oplus E_K(W_1\|1) \oplus \cdots \oplus E_K(W_{l-1}\|B'\|0^*\|(l-1))$



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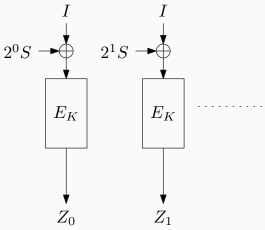

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$\widetilde{\text{XoP}[w]}$ PRF in Counter Mode

- $\widetilde{\text{XoP}[w]}$: XoP$[w]$ as used in CENC [Iwa06], and extended to include tweak
  - Introduction is new and comes with separate security proof
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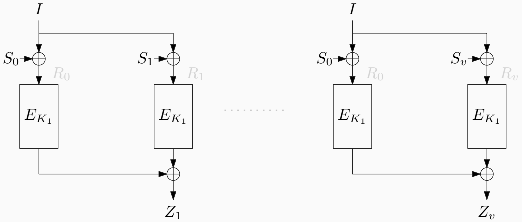

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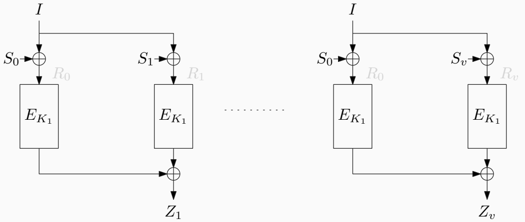
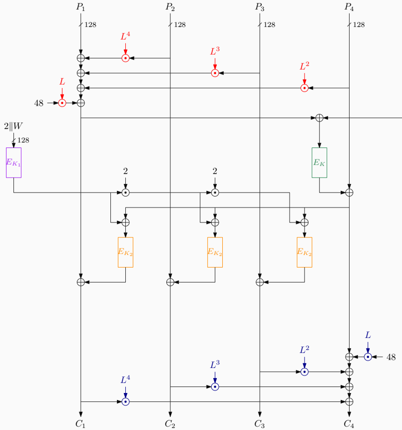
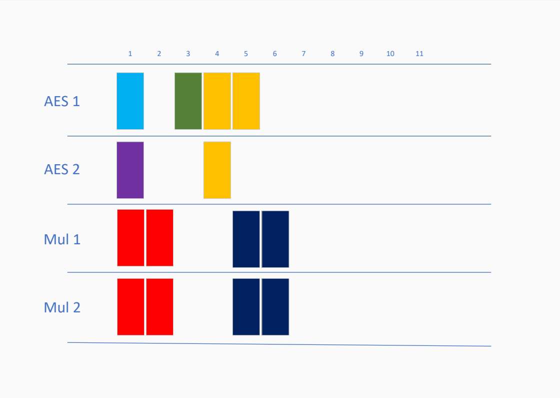


- Corresponding stream cipher runs $\widetilde{\text{XoP}[w]}$ in counter mode
- Stream cipher (and thus *bbb-ddd-AES*) is $2^{2n/3}$ PRF-secure when tweaks are not used too often

Implementation Design of $ddd\text{-}AES$ (512-Bit Message)





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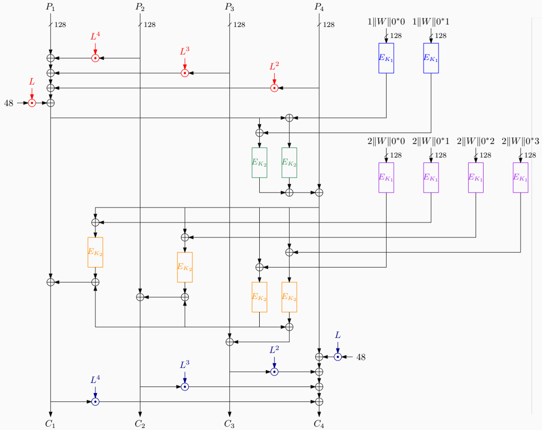

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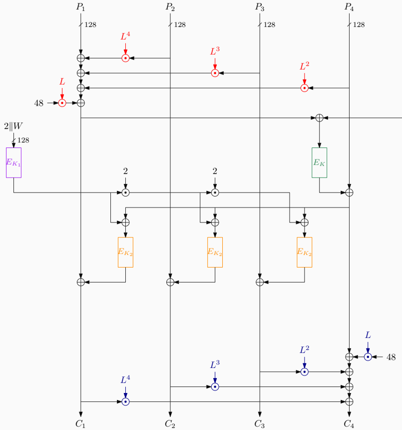

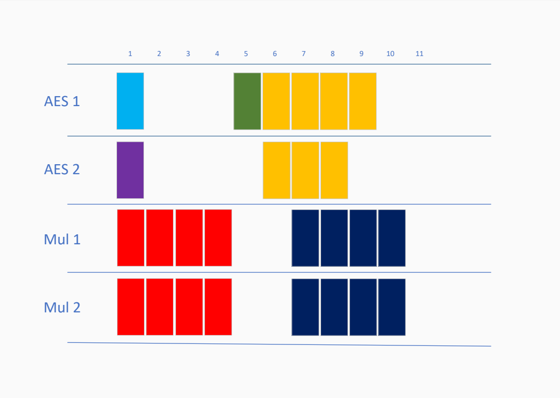

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