Introduction to Formal Reasoning 2019
Exam
(15/01/20)

Before you read on, write your name, student number and study on the answer sheet! This exam consists of eighteen exercises and each of these exercises is worth five points. The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

The following exercises are concerned with propositional logic:

1. Consider the formula of propositional logic:

$$a \vee b \wedge c \leftrightarrow a \vee b \wedge a \vee c$$

   (a) Write this formula according to the official grammar from the course notes.

   (b) Give the truth table of this formula.

2. Consider the following English sentence:

   I'll be damned if I don't understand this.

   (a) Give a formula of propositional logic that formalizes the meaning of this sentence using the dictionary:

D	I am damned
U	I understand this

   (b) Explain why this sentence expresses that I understand this.

3. Consider the model v_3 of propositional logic with $v_3(a) = 1$ and $v_3(b) = 0$. Explain what corresponds in a truth table with this model.

The following exercises are concerned with predicate logic:

4. Consider the following English sentence:

   Nobody likes everything.

   Give a formula of predicate logic that formalizes the meaning of this sentence using the dictionary:

H	the domain of people
T	the domain of things
$L(x, y)$	x likes y

5. Consider the following English sentence:

   There are exactly three countries in the world not using the metric system.

   Give a formula of predicate logic with equality that formalizes the meaning of this sentence using the dictionary:

C	the domain of countries in the world
$M(x)$	x uses the metric system

6. We are looking for a model of predicate logic, and an interpretation in this model that makes the following formula of predicate logic with equality true:

$$\exists y_1, y_2 \in X\,[y_1 \neq y_2] \land \forall x \in X\,\exists y_1, y_2 \in X\,[x \neq y_1 \land x \neq y_2]$$

   Give such a model and interpretation where the interpretation of X has a *minimal* number of elements. Explain your answer.

The following exercises are concerned with formal languages:

7. Give an infinite language L_7 (i.e., that contains infinitely many different words), such that

$$L_7{}^R \subseteq \overline{L_7}$$

   As a reminder: the reverse of a language is defined by:

$$L^R := \{w^R \mid w \in L\}$$

   Explain your answer.

8. Give a regular expression for the language:

$$L_8 := \{w \in \{a, b, c\}^* \mid w \text{ both contains } ab \text{ and } bc\}$$

9. Let be given the context-free grammar G_9:

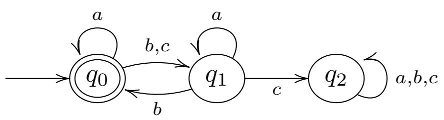
$$S \to aSb \mid \lambda$$

   Show that the following property is an invariant of this grammar:

$$P(w) := w \text{ contains the same number of } a\text{'s and } b\text{'s}$$

The following exercises are concerned with automata:

10. Give the right linear context-free grammar that corresponds to the deterministic finite automaton M_{10}:



   Make sure your grammar also has a non-terminal that corresponds to state q_2.

11. Give a deterministic finite automaton that recognizes the language:

$$L_{11} := \{w \in \{a, b, c\}^* \mid w \text{ does not contain } aab\}$$

12. Give a non-deterministic finite automaton with at most five states that recognizes the language:

$$L_{12} := \mathcal{L}((a \cup b \cup c)^* a(a \cup b \cup c)(a \cup b \cup c)(a \cup b \cup c))$$

The following exercises are concerned with discrete mathematics:

13. A graph is called *cubic* if every vertex has degree three. Give a connected, bridgeless, planar, cubic graph that contains a Hamiltonian cycle. Explain why your graph has all five requested properties.

14. We define a sequence $(a_n)_{n\geq 3}$ using the recursive equations:

$$\begin{aligned} a_3 &= 4 \\ a_{n+1} &= a_n + 2n + 1 \quad \text{for } n \geq 3 \end{aligned}$$

 (a) Show how to use this recursive definition to compute a_5.

 (b) Prove by induction that $a_n = n^2 - 5$ for all $n \geq 3$.

 (c) Use the previous sub-exercise to compute a_{45}.

15. We want to put four objects in two boxes, where boxes may be left empty. We will count the number of ways that this can be done in four different variants.

 In each variant giving the number of ways is sufficient, you do not need to explain how you obtained your answer. This means also that you do not need to relate your answers to binomial coefficients, Stirling numbers or Bell numbers.

 However, if you are insecure about whether you got the correct answer, explanations are allowed.

 (a) In how many ways can one put four objects in two boxes, where both the objects and the boxes are indistinguishable.

 (b) In how many ways can one put four objects in two boxes, where the objects are indistinguishable but the boxes are distinguishable.

 (c) In how many ways can one put four objects in two boxes, where the objects are distinguishable but the boxes are indistinguishable.

 (d) In how many ways can one put four objects in two boxes, where both the objects and the boxes are distinguishable.

The following exercises are concerned with modal logic:

16. (a) Explain the difference between epistemic logic and doxastic logic.

 (b) Give an axiom scheme that holds in one of these logics, but not in the other. Explain your answer. *Note:* An axiom scheme that holds is a formula that holds independent from the interpretation of the propositional variables. In particular, you don't have to know the name of your axiom scheme.

17. In the semantics of modal logic we use the symbols '$\Vdash$' and '$\vDash$'. Below there are eight different notations using these symbols. Write down in which case(s) the symbol is used correctly. *Note:* You don't have to explain what these notations mean.

$$\begin{array}{cc} \mathcal{M}, x_1 \Vdash f & \mathcal{M}, x_1 \vDash f \\ x_1 \Vdash f & x_1 \vDash f \\ \mathcal{M} \Vdash f & \mathcal{M} \vDash f \\ \Vdash f & \vDash f \end{array}$$

18. Consider the following English sentence:

    I will only worry about Formal Reasoning until I pass the exam.

    Give an LTL formula that formalizes the meaning of this sentence using the dictionary:

W	I worry about Formal Reasoning
E	I pass the exam of Formal Reasoning

    You may assume that the atomic formula E is true at exactly one moment in time.

    Hint: The sentence does not imply that before passing the exam you are constantly worrying (maybe at some times you are not thinking about it), just that you will stop worrying once you pass the exam.