Formal Reasoning 2019
Test Block 3: Discrete Mathematics and Modal Logic
(18/12/19)

Before you read on, write your name, student number and study on the answer sheet!

We will only look at scratch paper if it has your name on it and you refer to it on the answer sheet. If not, we prefer that you do not hand in your scratch paper.

The mark for this test is the number of points divided by ten. The first ten points are free. Good luck!

1. The *hypercube graph* Q_n has as its vertices the elements of $\{a, b\}^n$ (the words of length n over the alphabet $\{a, b\}$). Two vertices are connected by an edge when they differ in exactly one position. For example in Q_4 there is an edge between *aaab* and *abab*, but not between *aaab* and *abbb*. (15 points)

   Draw Q_0, Q_1, Q_2 and Q_3.

2. In Pascal's triangle, we have:

$$\binom{5}{3} = \binom{4}{2} + \binom{4}{3}$$

   (a) Give the values of these three binomial coefficients, and show that the equation holds. (10 points)

   (b) Give the list of selections from the set $\{1, 2, 3, 4, 5\}$ that corresponds to the first binomial coefficient, and divide that list in two sublists that correspond to the two other binomial coefficients. Explain how this division shows why this equation holds. (15 points)

3. We use the dictionary:

R	it rains
W	I am wet

   (a) Using a *temporal* interpretation for the modalities (i.e., interpreting '$\Box$' as 'always'), explain the difference in meaning between the two modal formulas: (15 points)

$$\Box(R \to W)$$
$$R \to \Box W$$

   (b) Which of these two formulas most accurately gives the meaning of the sentence: (10 points)

   When it rains, I always get wet.

   Explain your answer.

4. We define a sequence (a_n) using the recursion equations:

$$a_0 = 0$$
$$a_{n+1} = a_n + 9 \cdot 10^n \qquad \text{for } n \geq 0$$

   (a) Show how to compute a_3 from this definition. (10 points)

   (b) Prove using induction that $a_n = 10^n - 1$ for all $n \geq 0$. (15 points)