# Formal Reasoning 2019
# Test Blocks 1, 2 and 3: Additional Test
**(08/01/20)**

Before you read on, write your name, student number and study on the answer sheet! The mark for this test is the number of points divided by ten. The first ten points are free. For each (sub)question the maximum score is indicated. Good luck!

1. We can translate LTL formulas into predicate logic with equality using the following dictionary: (30 points)

| | |
|---|---|
| $T$ | domain of time instances |
| $F$ | domain of atomic propositions |
| $t_0$ | the time instance of the world $x_0$ |
| $a$ | atomic proposition $a$ |
| $b$ | atomic proposition $b$ |
| $B(i,j)$ | time $i$ is strictly before time $j$ |
| $X(i,j)$ | time $j$ is one step after time $i$ |
| $H(i,f)$ | atomic proposition $f$ is true at time $i$ |

For example $x_1 \Vdash \mathcal{G}a$ translates to:

$$\exists t_1 \in T\,[X(t_0,t_1) \wedge \forall i \in T\,[B(t_1,i) \vee t_1 = i \to H(i,a)]]$$

Give a translation in this style of:

$$x_1 \Vdash a\,\mathcal{U}\,b$$

2. Consider the context-free grammar $G_2$ : (30 points)

$$S \to A \mid bS$$
$$A \to aA \mid cS \mid \lambda$$

We can show that $ab \notin \mathcal{L}(G_2)$ by using the invariant:

$$P(w) := \big(w \text{ does not contain any of: } ab,\ aS,\ Ab,\ Sb,\ AS,\ SS\big)$$

To show that this is an invariant, you need to check that $P(S)$ holds (that is easy), but you also need to check that the property is preserved by each production rule in the grammar. The exercise is to show that this is the case for the rule:

$$S \to bS$$

(You do not need to show this for the other four rules.)

3. Consider the following equation:

$$\left\{ {n \atop n-2} \right\} = \binom{n}{3} + \frac{1}{2}\binom{n}{2}\binom{n-2}{2}$$

   (a) Show that this equation holds for $n = 4$ by computing the Stirling number and the binomial coefficients explicitly. (10 points)

   (b) Give a combinatorial argument that shows that this equation holds for all $n \geq 4$. (20 points)