Formal Reasoning 2020
Test Block 1: Propositional and Predicate Logic
(21/09/20)

There are ten multiple choice questions and one open question. Each multiple choice question is worth 8 points, and the open question is worth 10 points. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Propositional logic

1. We use the dictionary

C	there is a pandemic
M	this test has multiple choice questions

Which of the following formulas of propositional logic is the best formalization of the following English sentence:

> *This test has multiple choice questions, both when there is a pandemic and when there is not a pandemic.*

(a) $(C \to M) \land (\neg C \to M)$
(b) $(C \to M) \lor (\neg C \to M)$
(c) $(C \land \neg C) \leftrightarrow M$
(d) $(C \land \neg C) \to M$

1. We use the dictionary

C	there is a pandemic
M	this test has multiple choice questions

Which of the following formulas of propositional logic is the best formalization of the following English sentence:

> *This test has multiple choice questions, because there is a pandemic.*

(a) $M \to C$
(b) $C \to M$
(c) $C \leftrightarrow M$
(d) $C \land M$

2. The truth table of the formula of propositional logic

$$a \land \neg b \leftrightarrow c$$

consists of eight rows. How many of those rows have a 0?

(a) 1

(b) 2
(c) 4
(d) None of the above.

2. The truth table of the formula of propositional logic

$$a \wedge \neg b \vee c$$

consists of eight rows. How many of those rows have a 0?

(a) 1
(b) 2
(c) 4
(d) None of the above.

3. A model of propositional logic corresponds in the truth table to . . .

(a) One of the rows.
(b) One of the columns.
(c) The final column for the whole formula.
(d) The whole table.

3. How many rows are there in the truth table of a propositional formula that contains only a single atomic proposition a?

(a) 1
(b) 2
(c) 4
(d) It depends on the formula.

4. Does the following hold?

$$\neg(a \to b) \equiv \neg a \to \neg b$$

(a) Yes, this is one of the distributive laws.
(b) Yes, this is the law of contraposition.
(c) No, you can see this by comparing two columns in an appropriate truth table.
(d) No, because there is a model in which this logical equivalence is not true.

4. Does the following hold?

$$\neg(a \to b) \models \neg a \to \neg b$$

(a) Yes, this follows from one of the distributive laws.
(b) Yes, you can see this by comparing two columns in an appropriate truth table.
(c) No, because the law of contraposition gives that $a \to b \models \neg b \to \neg a$ instead of $a \to b \models \neg a \to \neg b$.

(d) No, because there is a model in which this logical consequence is not true.

5. Which of the following holds?

   (a) If $\models f \vee g$ then $\models f$ or $\models g$ (or both).
   (b) If $f \vee g \equiv h$ then $f \equiv h$ or $g \equiv h$ (or both).
   (c) If $\models f$ or $\models g$ (or both) then $\models f \vee g$.
   (d) If $f \equiv h$ or $g \equiv h$ (or both) then $f \vee g \equiv h$.

5. Which of the following does *not* hold?

   (a) If $\models f \wedge g$ then $\models f$ and $\models g$.
   (b) If $f \wedge g \equiv h$ then $f \equiv h$ and $g \equiv h$.
   (c) If $\models f$ and $\models g$ then $\models f \wedge g$.
   (d) If $f \equiv h$ and $g \equiv h$ then $f \wedge g \equiv h$.

Predicate logic

6. What is the syntax of the following formula according to the official grammar from the course notes?

$$(\exists x_1, x_2 \in D\,(x_1 \neq x_2))$$

   (a) $(\exists x_1 \in D\,(\exists x_2 \in D\,(\neg(x_1 = x_2))))$
   (b) $(\exists x_1 \in D\,(\exists x_2 \in D\,\neg(x_1 = x_2)))$
   (c) $(\exists x_1 \in D\,[\exists x_2 \in D\,[\neg(x_1 = x_2)]])$
   (d) $\exists x_1 \in D\,[\exists x_2 \in D\,[\neg(x_1 = x_2)]]$

6. What is the syntax of the following formula according to the official grammar from the course notes?

$$\forall x \in D\,[\exists y \in D\,R(x,y) \wedge P(x)]$$

   (a) $\forall x \in D\,[\exists y \in D\,[(R(x,y) \wedge P(x))]]$
   (b) $\forall x \in D\,[(\exists y \in D\,[R(x,y)] \wedge P(x))]$
   (c) $(\forall x \in D\,(\exists y \in D\,(R(x,y) \wedge P(x))))$
   (d) $(\forall x \in D\,((\exists y \in D\,R(x,y)) \wedge P(x)))$

7. We use the dictionary

P	domain of people
V	domain of vehicles
$F(x)$	x wears a face mask
$T(x)$	x is public transport vehicle
$I(x,y)$	x is inside y

Which of the following formulas of predicate logic is the best formalization of the following English sentence:

In public transport, people wear a face mask.

(a) $\forall x \in P\, \forall y \in V\, (T(y) \wedge I(x, y) \rightarrow F(x))$
(b) $\forall x \in P\, \forall y \in V\, (T(y) \wedge I(x, y) \wedge F(x))$
(c) $\forall x \in P\, \exists y \in V\, (T(y) \wedge I(x, y) \wedge F(x))$
(d) $\forall x \in P\, ((\forall y \in V\, (T(y) \wedge I(x, y))) \rightarrow F(x))$

7. We use the dictionary

P	domain of people
V	domain of vehicles
$F(x)$	x wears a face mask
$T(x)$	x is public transport vehicle
$I(x, y)$	x is inside y

Which of the following formulas of predicate logic is the best formalization of the following English sentence:

If a person does not wear a face mask, he or she is not traveling by public transport.

(a) $\forall x \in P\, (\neg F(x) \rightarrow \neg \forall y \in V(T(y) \wedge I(x, y)))$
(b) $\forall x \in P\, (\neg F(x) \rightarrow \neg \exists y \in V(T(y) \wedge I(x, y)))$
(c) $\exists x \in P\, (\neg F(x) \rightarrow \neg \forall y \in V(T(y) \wedge I(x, y)))$
(d) $(\exists x \in P\, \neg F(x)) \rightarrow \neg \exists y \in V(T(y) \wedge I(x, y))$

8. How does one express in predicate logic with equality, that a domain D has exactly two elements that satisfy a predicate P?

(a) $\exists x_1, x_2 \in D\, [x_1 \neq x_2 \wedge P(x_1) \wedge P(x_2)]$
(b) $\exists x_1, x_2 \in D\, [x_1 \neq x_2 \wedge \forall y \in D\, [P(y) \leftrightarrow y = x_1 \vee y = x_2]]$
(c) $\exists x_1, x_2 \in D\, [x_1 \neq x_2 \wedge \forall y \in D\, [P(y) \rightarrow y = x_1 \vee y = x_2]]$
(d) $\exists x_1, x_2 \in D\, [x_1 \neq x_2] \wedge \forall y \in D\, [P(y) \rightarrow y = x_1 \vee y = x_2]$

8. How does one express in predicate logic with equality, that a domain D does not have exactly one element that satisfies a predicate P (i.e., there are no elements for which P holds, or two or more)?

(a) $\forall x \in D[P(x) \rightarrow \exists y \in D[P(y)]]$
(b) $\forall x, y \in D[P(x) \wedge P(y) \rightarrow x \neq y]$
(c) $(\neg \exists x \in D[P(x)]) \vee (\exists x_1, x_2 \in D[P(x_1) \wedge P(x_2)])$
(d) $\neg \exists x \in D[P(x) \wedge \forall y \in D[P(y) \rightarrow x = y]]$

9. We define a model

$$M = (\mathbb{N}, <)$$

and an interpretation I that maps N to $\mathbb{N}$ and $L(x, y)$ to $x < y$. Which of the following statements hold?

(a) $(M, I) \models \exists x \in N\, \forall y \in N\, L(x, y)$
(b) $(M, I) \models \exists x \in N\, \forall y \in N\, L(y, x)$
(c) $(M, I) \models \forall x \in N\, \exists y \in N\, L(x, y)$

(d) $(M, I) \vDash \forall x \in N\, \exists y \in N\, L(y, x)$

9. We define a model

$$M = (\mathbb{N}, \leq)$$

and an interpretation I that maps N to $\mathbb{N}$ and $L(x, y)$ to $x \leq y$. Which of the following statements does *not* hold?

(a) $(M, I) \vDash \exists x \in N\, \forall y \in N\, L(x, y)$

(b) $(M, I) \vDash \exists x \in N\, \forall y \in N\, L(y, x)$

(c) $(M, I) \vDash \forall x \in N\, \exists y \in N\, L(x, y)$

(d) $(M, I) \vDash \forall x \in N\, \exists y \in N\, L(y, x)$

10. Which of the following statements is correct?

(a) $\neg\forall x \in D\, (P(x) \to Q(x)) \equiv \forall x \in D\, (\neg P(x) \to \neg Q(x))$

(b) $\neg\forall x \in D\, (P(x) \to Q(x)) \equiv \forall x \in D\, (\neg(P(x) \to Q(x)))$

(c) $\neg\forall x \in D\, (P(x) \to Q(x)) \equiv \exists x \in D\, (P(x) \wedge \neg Q(x))$

(d) $\neg\forall x \in D\, (P(x) \to Q(x)) \equiv \exists x \in D\, (\neg P(x) \wedge \neg Q(x))$

10. Which of the following statements is correct?

(a) $\neg\exists x \in D\, (P(x) \wedge Q(x)) \equiv \exists x \in D\, (\neg P(x) \wedge \neg Q(x))$

(b) $\neg\exists x \in D\, (P(x) \wedge Q(x)) \equiv \exists x \in D\, (\neg(P(x) \wedge Q(x)))$

(c) $\neg\exists x \in D\, (P(x) \wedge Q(x)) \equiv \forall x \in D\, (P(x) \to \neg Q(x))$

(d) $\neg\exists x \in D\, (P(x) \wedge Q(x)) \equiv \forall x \in D\, (\neg P(x) \to \neg Q(x))$

Open question

11. Give an interpretation in a model that makes the following formula true:

$$(\exists x \in D\;\; R(x, x)) \wedge \forall y_1, y_2 \in D\, \exists z \in D\, (R(y_1, z) \wedge R(z, y_2))$$

Use an interpretation for D that is a set of numbers.

You do *not* need to explain why your model makes this formula true.

11. Give an interpretation in a model that makes the following formula true:

$$(\exists x \in D\, \neg R(x, x)) \wedge \forall y \in D\, (\exists z_1 \in D\, R(y, z_1) \wedge \exists z_2 \in D\, R(z_2, y))$$

Use an interpretation for D that is a set of numbers.

You do *not* need to explain why your model makes this formula true.