Formal Reasoning 2020
Test Block 2: Languages and Automata
(26/10/20)

There are five multiple choice questions and three open questions. Each multiple choice question is worth 9 points, and the open questions are worth 15 points. The mark for this test is the number of points divided by ten, and the first ten points are free. Good luck!

Multiple choice questions

1. Which of the following four languages is different from the other three?
   (a) $\emptyset$
   (b) $\emptyset^*$
   (c) $\{\lambda\}$
   (d) $\{\lambda\}^*$

1. Which of the following four languages is different from the other three?
   (a) $(\{a, b\})^*$
   (b) $(\{b, a, a\})^*$
   (c) $(\{a\}^* \cup \{b\}^*)$
   (d) $(\{a\} \cup \{b\})^*$

2. Is the intersection of two regular languages always regular?
   (a) Yes, because if all words in both languages are regular, then the words in the intersection are also certainly regular.
   (b) Yes, because the complement of a regular language is always regular, and $L \cap L' = \overline{\overline{L} \cup \overline{L'}}$.
   (c) No, because the intersection symbol '$\cap$' is not allowed in a regular expression.
   (d) No, because a sublanguage of a regular language does not need to be regular.

2. Is the complement of a regular language always regular?
   (a) Yes, because the regular languages are exactly the languages that can be recognized with a DFA, and it is easy to construct a DFA for the complement.
   (b) Yes, because the complement of the complement is the language itself, so no information is lost by complementation.
   (c) No, because the complement symbol '$\overline{L}$' is not allowed in a regular expression.
   (d) No, because if the words in a language are regular, all words that are not regular will be in the complement.

3. Consider the context-free grammar G_3:

$$S \to aS \mid Aa$$
$$A \to bS \mid Aa \mid \lambda$$

We want to show that $ba \notin \mathcal{L}(G_3)$ and for this we consider the following property as an invariant:

$$P(w) := w \text{ contains at least one } S \text{ or one } a$$

Does that work?

(a) Yes, because after the first production step the words in a production will contain an a, which will not go away anymore in the rest of the production.

(b) Yes, because the property holds for S, and in each production step for S the symbol a is introduced.

(c) No, because the invariant is useless for this.

(d) No, because we have the rule $A \to \lambda$, and this rule does not introduce either an S or an a.

3. Consider the context-free grammar G_3:

$$S \to aS \mid Aa$$
$$A \to bS \mid Aa \mid \lambda$$

We want to show that $ba \notin \mathcal{L}(G_3)$ and for this consider the following property as an invariant:

$$P(w) := \text{if } w \text{ contains the symbol } b \text{ then it contains at least two } a\text{'s}$$

Does that work?

(a) Yes, because to get the symbol b in a word, you need to use the rule $A \to bS$, and any production that involves that rule needs to go through the rule $S \to Aa$ twice.

(b) Yes, because this property holds for all words in the language of G_3.

(c) No, because the invariant is useless for this.

(d) No, because we have the rule $A \to bS$.

4. Is there for all context-free grammars a right linear grammar that produces the same language?

(a) Yes, you can find such a grammar by converting to an automaton, and then converting back to a grammar.

(b) Yes, because all right linear grammars are context-free grammars.

(c) No, because not all context-free languages are regular.

(d) No, because some rules can violate the condition of the context-free grammars being right linear.

4. Is there for all non-deterministic finite automata a deterministic finite automaton that recognizes the same language?

(a) Yes, you can construct such an automaton using the power set construction.

(b) Yes, because you are not required to have non-determinism in a non-deterministic automaton.

(c) No, because in a non-deterministic automaton λ transitions are allowed.

(d) No, because non-deterministic automata are more efficient than deterministic automata.

5. Are there languages for which every deterministic finite automaton has a sink (a non-final state for which each transition will loop back to that state)?

   (a) Yes, when there is a word w for which any extension ww' will not be in the language.

   (b) Yes, because each state needs to have a transition for each symbol.

   (c) No, because you can always split a sink in multiple states.

   (d) No, because you can always use a non-deterministic automaton.

5. Are there languages for which every non-deterministic finite automaton does *not* have a sink (a non-final state for which each transition will loop back to that state)?

   (a) Yes, in the case that each word w has an extension ww' that is in the language, there cannot be a sink.

   (b) Yes, in non-deterministic automata there never are sinks.

   (c) No, one can always add a sink to an automaton without changing the language that is accepted.

   (d) No, there are languages for which each non-deterministic finite automaton has a sink.

Open questions

6. Give a regular expression for the language:

$$L_6 := \{w \in \{a, b\}^* \mid w \text{ does not contain } aab\}$$

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7. Is the following language context-free?

$$L_7 := \{a^n b^n c^m \mid n, m \geq 0\}$$

   If so, give a context-free grammar.

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   If so, give a context-free grammar.

8. Give a deterministic finite automaton with the least number of states for the language:

$$L_8 := \{w \in \{a, b\}^* \mid \text{the number of } a\text{'s in } w \text{ is even iff the number of } b\text{'s is even}\}$$

(The abbreviation ‘iff’ stands for ‘if and only if’.)

8. Give a deterministic finite automaton with the least number of states for the language:

$$L_8 := \{w \in \{a, b\}^* \mid \text{the number of } a\text{'s in } w \text{ is odd iff the number of } b\text{'s is odd}\}$$

(The abbreviation ‘iff’ stands for ‘if and only if’.)