Formal Reasoning 2020
Test Blocks 1, 2 and 3: Additional Test
(16/12/20)

There are six multiple choice questions and two open questions. The open questions will be at the end of the test. Each multiple choice question is worth 10 points, and the open questions are worth 15 points. The first ten points are free. Good luck!

Multiple choice questions

1. The *exclusive or* operation in propositional logic, with symbol $\oplus$, is defined by the following truth table:

a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

The formula $a \oplus b$ should be read as 'a, or b, but not both'.

Which of the following formulas is *not* logically equivalent to $a \oplus b$?

(a) $a \land (\neg b) \lor (\neg a) \land b$

(b) $a \lor b \land \neg(a \land b)$

(c) $\neg a \leftrightarrow b$

(d) $\neg(a \leftrightarrow b)$

1. The *Sheffer stroke* operation in propositional logic, with symbol $|$, is defined by the following truth table:

a	b	$a \mid b$
0	0	1
0	1	1
1	0	1
1	1	0

This is also known as the *nand* operation, as it corresponds to the *nand*-gate which is one of the basic gates in logical circuits in a computer.

Which of the following formulas is *not* logically equivalent to $a \mid b$?

(a) $\neg(a \land b)$

(b) $\neg a \land \neg b$

(c) $a \to \neg b$

(d) $b \to \neg a$

2. Consider the model $M := (\mathbb{N}, 0, +)$ and the interpretation I defined by:

N	$\mathbb{N}$
o	0
$A(x, y, z)$	$x + y = z$

Which of the following statements does not hold?

(a) $(M, I) \models \forall x \in N\, \exists y \in N\, A(o, x, y)$
(b) $(M, I) \models \forall x \in N\, \exists y \in N\, A(x, o, y)$
(c) $(M, I) \models \forall x \in N\, \exists y \in N\, A(y, o, x)$
(d) $(M, I) \models \forall x \in N\, \exists y \in N\, A(x, y, o)$

2. Consider the model $M := (\mathbb{N}, -)$ and the interpretation I defined by:

N	$\mathbb{N}$
$S(x, y, z)$	$x - y = z$

Which of the following statements does not hold?

(a) $(M, I) \models \forall x \in N\, \exists y \in N\, S(x, y, y)$
(b) $(M, I) \models \forall x \in N\, \exists y \in N\, S(x, x, y)$
(c) $(M, I) \models \forall x \in N\, \exists y \in N\, S(x, y, x)$
(d) $(M, I) \models \forall x \in N\, \exists y \in N\, S(y, x, x)$

3. If for a language L is given that $L^* = L$, what does *not* necessarily follow?

(a) $LL = L$
(b) $\lambda \in L$
(c) L is infinite
(d) L is non-empty

3. If for a language L is given that $L^R = L$, what *does* necessarily follow?

(a) if $w \in L^*$ then also $w^R \in L^*$
(b) there is a $w \in L$ for which also $w^R \in L$
(c) $\lambda \in L$, because $\lambda^R = \lambda$
(d) there is a $w \in L$ with $w^R = w$

4. Let be given a deterministic finite automaton $M := \langle \Sigma, Q, q_0, F, \delta \rangle$, with $F \neq Q$. What do we know?

(a) $\lambda \notin L(M)$
(b) M has a sink
(c) $q_0 \notin F$
(d) none of the above

4. Let be given a deterministic finite automaton $M := \langle \Sigma, Q, q_0, F, \delta \rangle$, with $\lambda \notin L(M)$. What does *not* follow?

(a) $F \neq Q$
(b) M has a sink

(c) $q_0 \notin F$

(d) none of the above

5. The four color theorem says that planar graphs always have a chromatic number that is not higher than four. Is the converse (each graph with chromatic number not higher than four is always planar) also true?

   (a) No, the graph $K_{3,3}$ is not planar, but it has chromatic number two.

   (b) Yes, the graph K_4 has chromatic number four, and is planar.

   (c) No, the graph K_4 has chromatic number four, but can be drawn with crossing edges.

   (d) Yes, the graph K_5 has chromatic number five, and is not planar.

5. Euler's theorem is stated in the course notes about connected graphs with at least two vertices. Are both of these conditions necessary?

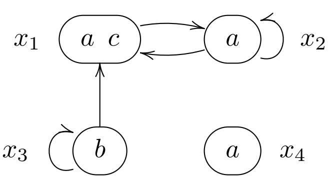
   (a) Yes, because there cannot be an Eulerian path if the graph is not connected or has at most one vertex, no matter what the degrees are.

   (b) Yes, but for certain graphs there still can be an Eulerian path, even if the graph is not connected or has at most one vertex.

   (c) No, there are no connected graphs with less than two vertices, because then there cannot be a path in the graph, so the requirement on the number of vertices is not necessary.

   (d) No, all graphs with less than two vertices are connected, so the requirement on the number of vertices is not necessary.

6. Consider the Kripke model $\mathcal{M}$:



In which worlds does the formula $\Box\Diamond c$ hold?

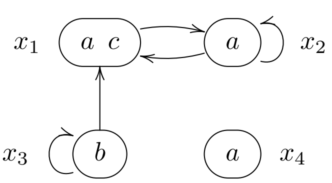
   (a) x_1 and x_2

   (b) x_1 and x_4

   (c) only x_1

   (d) none

6. Consider the Kripke model $\mathcal{M}$:



In which worlds does the formula $\Diamond\Box c$ hold?

(a) x_1 and x_2

(b) x_1 and x_4

(c) only x_1

(d) none

Open questions

7. Translate into a formula of predicate logic:

Grass is green, but grass is not the only green plant.

Use the dictionary:

P	the domain of plants
$G(x)$	x is a grass
$V(x)$	x is green

7. Give a regular expression for the language:

$$\{w \in \{a, b\}^* \mid w \text{ contains } ab, \text{ but } w \text{ does not contain } ba\}$$

7. Describe an LTL Kripke model in which the following LTL formula is true:

$$\neg(a \, \mathcal{U} \, b)$$

8. Write the following propositional formula according to the official grammar from the course notes, and give the full truth table:

$$\neg(\neg(((\neg a) \to a) \to a))$$

8. Give a non-deterministic finite automaton for the language

$$\{a\} \cup \{ab^n \mid n \text{ is odd}\}$$

with at most four states.

8. We define recursively:

$$a_0 = 1$$
$$a_{n+1} = 2a_n - n \qquad \text{for } n \geq 0$$

Prove by induction that $a_n = n + 1$ for all $n \geq 0$.