Later Credits: Reducing the Proof Cost of Step-Indexing in Iris

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MFoCS Seminar

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Introduction

- Later Credits: A new technique for program verification in the context of *step-indexed separation logics*

Later Credits: Resourceful Reasoning for the Later Modality

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In the past two decades, step-indexed logical relations and separation logics have both come to play a major role in semantics and verification research. More recently, they have been married together in the form of *step-indexed separation logics* like VST, iCAP, and Iris, which provide powerful tools for (among other things) building semantic models of richly typed languages like Rust. In these logics, propositions are given semantics using a step-indexed model, and step-indexed reasoning is reflected into the logic through the so-called "later" modality. On the one hand, this modality provides an elegant, high-level account of step-indexed reasoning; on the other hand, when used in sufficiently sophisticated ways, it can become a nuisance, turning perfectly natural proof strategies into dead ends.

In this work, we introduce *later credits*, a new technique for escaping later-modality quagmires. By leveraging the second ancestor of these logics—separation logic—later credits turn "the right to eliminate a later" into an ownable resource, which is subject to all the traditional modular reasoning principles of separation logic. We develop the theory of later credits in the context of Iris, and present several challenging examples of proofs and proof patterns which were previously not possible in Iris but are now possible due to later credits.

100

CCS Concepts: • **Theory of computation** → **Separation logic**; **Logic and verification**.

Additional Key Words and Phrases: Separation logic, Iris, step-indexing, later modality, transfinite

ACM Reference Format:

Contents

Contents

A Counter API

- API with methods for creating, incrementing and reading a counter:

```
new() := ref(0)

inc(p) :=
  let x = !p in
    p ← x + 1

get(p) := !p
```

A Counter API

- Some client code...

```
let c = new() in
  (inc(c) || inc(c))
  get(c)
```

- ...and the API code

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- Question: What is the result of `get(c)` in the client code?

Making the API Thread-Safe

How can we make the API thread-safe?

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Using the compare-and-set primitive: **`CAS`**(p, x, y)

- Semantics: *If value pointed to by p is equal to x, write y to p and return* **`true`**, *otherwise return* **`false`**

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new() := ref(0)

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    if ...
    then ...
    else ...

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```

Making the API Thread-Safe

How can we make the API thread-safe?

Using the compare-and-set primitive: **CAS**(p, x, y)

- Semantics: *If value pointed to by p is equal to x, write y to p and return* **true**, *otherwise return* **false**

```
new() := ref(0)

inc(p) :=
  let x = !p in
    if CAS(p, x, x + 1)
    then ()
    else inc(p)

get(p) := !p
```

Properties of the Counter API

```
let c = new() in
  (inc(c) || inc(c))
  get(c)
```

```
inc(p) :=
  let x = !p in
    if CAS(p, x, x + 1)
    then ()
    else inc(p)
```

The counter is now thread-safe:

- Calls to the API methods do not get stuck
- If calls to the API methods terminate, they yield the expected value

What have we seen so far?

- Assuming we wanted to verify thread-safety of the counter API:

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Reasoning about concurrency	*Concurrent* separation logic

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Reasoning about computation state	Hoare logic
Modular reasoning about resources	Separation logic
Reasoning about concurrency	*Concurrent* separation logic
Non-structural recursion	Step-indexing

What is Iris?

Step-indexed, concurrent separation logic framework

- Implemented in Coq
- Modularity: Specifications are reusable and composable
- Language-independent (Rust, OCaml, Scala, Go, ...)

Some Background on Iris

- Before discussing later credits, we need some background on Iris:

Iris from the ground up

A modular foundation for higher-order concurrent separation logic

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Abstract

Iris is a framework for higher-order concurrent separation logic, which has been implemented in the Coq proof assistant and deployed very effectively in a wide variety of verification projects. Iris was designed with the express goal of simplifying and consolidating the foundations of modern separation logics, but it has evolved over time, and the design and semantic foundations of Iris itself have yet to be fully written down and explained together properly in one place. Here, we attempt to fill this gap, presenting a reasonably complete picture of the latest version of Iris (version 3.1), from first principles and in one coherent narrative.

Propositions in Iris

P, Q	$::=$	$\mathsf{False} \quad \mid \quad \mathsf{True}$	*Boolean values*
	$\mid$	$\ldots$	
	$\mid$	$P \lor Q$	*Disjunction*
	$\mid$	$\exists x : \tau.\ P$	*Ex. quantification*
	$\mid$	$\forall x : \tau.\ P$	*Univ. quantification*
	$\mid$	$\{P\}\ e\ \{v.\ Q\}$	*Hoare triples*
	$\mid$	$P * Q$	*Separating conjunction*
	$\mid$	$\ell \mapsto v$	*Points-to assertion*
	$\mid$	$\triangleright P$	*Later modality*
	$\mid$	$\ldots$	

Propositions in Iris

$$
\begin{array}{rcll}
P, Q & ::= & \mathsf{False} \mid \mathsf{True} & \textit{Boolean values} \\
& \mid & \ldots & \\
& \mid & P \vee Q & \textit{Disjunction} \\
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& \mid & \{P\}\ e\ \{v.\ Q\} & \textit{Hoare triples} \\
& \mid & P * Q & \textit{Separating conjunction} \\
& \mid & \ell \mapsto v & \textit{Points-to assertion} \\
& \mid & \triangleright P & \textit{Later modality} \\
& \mid & \ldots &
\end{array}
$$

- Higher-order logic: Quantifiers can range over any type
- Hoare triples are first-order

Hoare Logic

Hoare triples express partial program correctness:

$$\{P\}\ e\ \{v.\ Q\}$$

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$$\{P\}\ e\ \{v.\ Q\}$$

For an initial state satisfying *precondition* P:

- Execution of e does not crash
- If e terminates with value v', the final state satisfies the *postcondition* Q by $Q[v \leftarrow v']$

Separation Logic

- Propositions convey *ownership* of state

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Separation Logic

- Propositions convey *ownership* of state
- Points-to assertion: $\ell \mapsto v$
- Separating conjunction: $P * Q$
  - $(\ell \mapsto v) * (k \mapsto w)$ implies $\ell \neq k$
- Frame rule:

$$\textsc{Frame}\quad \frac{\{P\}\ e\ \{v.\ Q\}}{\{P * R\}\ e\ \{v.\ (Q * R)\}}$$

Concurrent Separation Logic

- Parallel composition rule of CSL:

$$\textsc{Par}\quad \frac{\{P_1\}\; e_1\; \{v.\; Q_1\} \qquad \{P_2\}\; e_2\; \{v.\; Q_2\}}{\{P_1 * P_2\}\;\; e_1 \,||\, e_2\;\; \{v.\; Q_1 * Q_2\}}$$

Contents

Contents

Step-Indexing

Technique for reasoning about (languages with) “cyclic” features

- E.g. higher-order state:

$$
\begin{aligned}
&r \leftarrow \lambda x.\, (!\, r)\ x\,; \\
&(!\, r)\ ()
\end{aligned}
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Step-Indexing

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Step-Indexing

Technique for reasoning about (languages with) “cyclic” features

- E.g. higher-order state:

$$r \leftarrow \lambda x.\ (!\,r)\ x;$$
$$(!\,r)\ ()$$

- Problem: Naive models are unsound
- Solution: *Stratify* the model with step-indices
  - Expression e satisfies predicate P with n steps “on the clock” if, when e reduces to e' in $i < n$ steps, then e' satisfies P with $n - i$ steps left “on the clock”

A Minimal Step-Indexed Logic

- Expression grammar:

$$
\begin{array}{rclclr}
P, Q & ::= & \mathsf{False} & \mid & \mathsf{True} & \textit{Boolean values} \\
 & \mid & P \wedge Q & & & \textit{Conjunction} \\
 & \mid & P \vee Q & & & \textit{Disjunction} \\
 & \mid & P \Rightarrow Q & & & \textit{Implication} \\
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$$SProp \triangleq \{A \subseteq \mathbb{N} \mid \forall n \geq m.\ \ n \in A \Rightarrow m \in A\} = \mathcal{P}^{\downarrow}(\mathbb{N})$$

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$$[\![\text{False}]\!] \triangleq \emptyset \qquad [\![P \wedge Q]\!] \triangleq [\![P]\!] \cap [\![Q]\!]$$

$$[\![\text{True}]\!] \triangleq \mathbb{N} \qquad [\![P \vee Q]\!] \triangleq [\![P]\!] \cup [\![Q]\!]$$

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$$[\![\forall x.P]\!]_\rho \triangleq \bigcap_v [\![P]\!]_{\rho[x \leftarrow v]} \qquad [\![x]\!]_\rho \triangleq \rho(x)$$

$$[\![\exists x.P]\!]_\rho \triangleq \bigcup_v [\![P]\!]_{\rho[x \leftarrow v]}$$

A Minimal Step-Indexed Logic

- A selection of rules:

$$P \vdash \text{True} \qquad \text{False} \vdash P$$

$$\frac{P \qquad Q}{P \wedge Q} \qquad P \wedge Q \vdash P \qquad P \wedge Q \vdash Q$$

$$P \vdash P \vee Q \qquad Q \vdash P \vee Q \qquad \frac{P \vdash R \qquad Q \vdash R}{P \vee Q \vdash R}$$

- Nothing new or exciting so far...

The Later Modality

- Updated expression grammar:

P, Q	$::=$	False | True		*Boolean values*
	|	$P \wedge Q$		*Conjunction*
	|	$P \vee Q$		*Disjunction*
	|	$P \Rightarrow Q$		*Implication*
	|	$\exists x : \tau.\ P$		*Ex. quantification*
	|	$\forall x : \tau.\ P$		*Univ. quantification*
	|	$\triangleright P$		*Later modality*

The Later Modality

- Updated semantic model:

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$$[\![\exists x.P]\!]_\rho \triangleq \bigcup_v [\![P]\!]_{\rho[x \leftarrow v]}$$

$$[\![\triangleright P]\!]_\rho \triangleq \{n \mid n = 0 \vee (n-1) \in [\![P]\!]_\rho\}$$

The Later Modality

- Rules:

LaterIntro

$$P \vdash \triangleright P$$

LaterMono

$$\frac{P \vdash Q}{\triangleright P \vdash \triangleright Q}$$

Löb

$$(\triangleright P \Rightarrow P) \vdash P$$

Later Elimination

- The Later Elimination Problem:
  $\triangleright P$ is in the context, but P is necessary to proceed,
  i.e. the guarding later must be *eliminated*

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- Three common techniques:
  - Timelessness of propositions

If P is timeless, a later modality guarding P can be “stripped away”.
A proposition P is timeless, *iff*

$$\forall n.\ \ 0 \in [\![P]\!] \Rightarrow n \in [\![P]\!].$$

Later Elimination

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If P is timeless, a later modality guarding P can be "stripped away".

$$\textsc{Timeless}\quad \frac{\{P * Q\}\ e\ \{v.\ R\} \qquad \text{timeless}(P)}{\{\triangleright P * Q\}\ e\ \{v.\ R\}}$$

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  - Commuting rules

LaterSep
$$\triangleright(P * Q) \dashv\vdash \triangleright P * \triangleright Q$$

LaterConj
$$\triangleright(P \vee Q) \dashv\vdash \triangleright P \vee \triangleright Q$$

LaterExist
$$\frac{\text{inhabited}(x)}{\triangleright(\exists x.\, \triangleright P) \dashv\vdash \exists x.\, \triangleright P}$$

Later Elimination

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  i.e. the guarding later must be *eliminated*

- Three common techniques:
  - Timelessness of propositions
  - Commuting rules
  - Program steps (PureStep)

$$\textsc{PureStep}\quad \frac{\{P\}\ e'\ \{v.\ Q\} \qquad e \rightarrow_{\text{pure}} e'}{\{\triangleright P\}\ e\ \{v.\ Q\}}$$

Idea: Amortisation of Later Eliminations

- Instead of one later elimination per step...

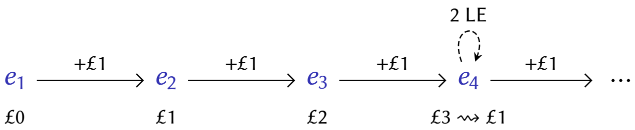
$$e_1 \xrightarrow{\text{1 LE}} e_2 \xrightarrow{\text{1 LE}} e_3 \xrightarrow{\text{1 LE}} e_4 \xrightarrow{\text{1 LE}} \cdots$$

Idea: Amortisation of Later Eliminations

- Instead of one later elimination per step...

$$e_1 \xrightarrow{\text{1 LE}} e_2 \xrightarrow{\text{1 LE}} e_3 \xrightarrow{\text{1 LE}} e_4 \xrightarrow{\text{1 LE}} \cdots$$

- ... generate one "token" per step which is redeemable for one later elimination, possibly at a *later* stage:



Contents

Contents

A Brief Interlude: Resources and the Update Modality

- *Resources* are a form of non-physical state, subject to the reasoning principles of separation logic

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UpdIntro
$$P \vdash \Rrightarrow P$$

UpdMono
$$\frac{P \vdash Q}{\Rrightarrow P \vdash \Rrightarrow Q}$$

UpdTrans
$$\Rrightarrow \Rrightarrow P \vdash \Rrightarrow P$$

UpdFrame
$$P * \Rrightarrow Q \vdash \Rrightarrow (P * Q)$$

UpdExec
$$\frac{\{P\}\ e\ \{v.\ Q\}}{\{\Rrightarrow P\}\ e\ \{v.\ Q\}}$$

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- Two main components of later credits in Iris:
  - New resource type $\pounds n$
  - New update modality ($\Rrightarrow_{\mathsf{le}}$) (*later elimination update*)

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  - New resource type $\pounds n$
  - New update modality ($\Rrightarrow_{\mathsf{le}}$) (*later elimination update*)

- $\pounds 1$ represents the "right to eliminate" one later

$$\textsc{PureStep}\quad \frac{\{P * \pounds 1\}\; e' \;\{v.\, Q\} \qquad e \rightarrow_{\mathsf{pure}} e'}{\{P\}\; e \;\{v.\, Q\}}$$

$$\textsc{LEUpdLater}\quad \pounds 1 * \triangleright P \vdash \Rrightarrow_{\mathsf{le}} P$$

$$\textsc{LEUpdExec}\quad \frac{\{P\}\; e \;\{v.\, Q\}}{\{\Rrightarrow_{\mathsf{le}} P\}\; e \;\{v.\, Q\}}$$

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PureStep
$$\frac{\{P * £1\}\ e'\ \{v.\ Q\} \qquad e \rightarrow_{\mathrm{pure}} e'}{\{P\}\ e\ \{v.\ Q\}}$$

LEUpdLater
$$£1 * \triangleright P \vdash \Rrightarrow_{\mathrm{le}} P$$

LEUpdExec
$$\frac{\{P\}\ e\ \{v.\ Q\}}{\{\Rrightarrow_{\mathrm{le}} P\}\ e\ \{v.\ Q\}}$$

- Facilitates reasoning about later eliminations as an ownable resource

CreditSplit
$$£(n+m) \Leftrightarrow £n * £m$$

Propositions in Iris

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	$\vert$	$P * Q$	*Separating conjunction*
	$\vert$	$\ell \mapsto v$	*Points-to connective*
	$\vert$	$\triangleright P$	*Later modality*
	$\vert$	$£n$	*Later credits*
	$\vert$	$\Rrightarrow_{\mathsf{le}} P$	*Later elimination update modality*
	$\vert$	$\ldots$	

Later Credits for Reordering Refinements

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$$(e_1 \mid\mid e_2) \leq_{\text{ctx}} (e_1, e_2)$$

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- ReLoC: Show contextual refinement using step-indexed logical relation
  - Right-hand program represented by *ghost state*

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- *Reordering refinement:* Execution order of e_1 and e_2 is not observable.

$$(e_1 \mid\mid e_2) \leq_{\text{ctx}} (e_1, e_2)$$

- ReLoC: Show contextual refinement using step-indexed logical relation
  - Right-hand program represented by *ghost state*
- In the paper: Reorderability of operations for JavaScript-style promises
  - Higher-order state!

Later Credits: Results

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- Step-indexing and the later modality are powerful tools for modelling and reasoning about type systems and languages with "cyclic features"...
- ...but they come at a price: The *proof burden* of challenging later eliminations.

Later Credits: Results

- Step-indexing and the later modality are powerful tools for modelling and reasoning about type systems and languages with "cyclic features"...
- ...but they come at a price: The *proof burden* of challenging later eliminations.

Later credits address this issue and facilitate:

- Simplifications of existing proofs
- New, previously unfeasible proofs

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