Combining uniqueness and linearity in one type system

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Papers

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  Dana Harrington
  TCP 2006

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Content

- Uniqueness and Linearity
  - What are these concepts + their use?
  - How do they relate?

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  - Uniqueness and linearity combined in typing rules from Marshall et al.

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  - Compare Harrington's uniqueness logic with linear logic
- Typing rules
  - Uniqueness and linearity combined in typing rules from Marshall et al.
- As programming language
  - Implement these typing rules in programming language
  - Performance

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- (Affine: used at most once)
- Example: a linear variable cake
- "You can't have your cake and eat it too"
- In Granule, where types are by default linear:

```
impossible : Cake -> (Happy, Cake)
impossible cake = (eat cake, have cake)
```

Use of linearity

- For example useful for file handling

```
twoChars : (Char, Char) <IO>
twoChars = let
h <- openHandle ReadMode "someFile";
(h, c1) <- readChar h;
(h, c2) <- readChar h;
() <- closeHandle h
in pure (c1, c2)
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- Must always have only 1 file handle for a certain file
- Must always close the file when done with it
- Each time, new file handle created after use, and each file handle is **used exactly once**

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- A unique variable must have **at most one reference** to it
- Different from affine types, which must be *used* at most once
- Subtle difference:
  - Affine variables may not be copied
  - There may not exist copies of unique variables

Example uniqueness

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- It's fine if you don't use the ticket though
- You can't sell a copy of your ticket to someone and still expect to be able to use it yourself at the cinema (it's only valid once)
- If all types are by default unique:

```
impossible : Ticket -> (Cash, Ticket)
impossible ticket = (sell ticket, ticket)
```

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- With lazy evaluation the order of actions is not always clear
- Example:

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a = [1,1,3,4]
f(a, writeArray(a,1,2))
```

- We want to prevent this

Use of uniqueness

- To fix this, you get a new array reference from read/write operations:

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a0 = [15,25,30]
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- If you know your reference is unique, you can directly update the array instead of having to copy the whole array
- Therefore, only allow **at most one reference**

Relation between linearity and uniqueness

- Uniqueness: constraint about past
  - Guarantees that the reference has not yet been duplicated

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- Consider the earlier example with mutable arrays
  - Linearity is too restrictive: discarding arrays is no problem
  - Linear (or affine) types are not strong enough: could have previously been a non-linear variable that was duplicated multiple times before being specialized to linear

Shared / non-linear variables

- Difference in how they relate to unrestricted variables

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  - Unique **can** be turned into shared by "forgetting" the constraint about their past
  - Shared **cannot** be turned into unique, since they might have been duplicated already

Paper: Uniqueness Logic

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- We will discuss some of the rules of this logic and compare them with linear logic

Comparison logic - introduction modalities

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Linear Logic

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Linear Logic | Uniqueness Logic

- introduction of ! on the left is unrestricted, while introduction of $\circ$ on the right is unrestricted

Comparison logic - contraction & weakening

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Linear Logic — Uniqueness Logic

- So linearity and uniqueness behave identically w.r.t. structural rules

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  - You don't have to copy an array to edit it if you know it's unique, then you can edit in-place
- They create a type system containing both unique and linear types (partly based on typing rules from Harrington)

Base system

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- The base system is linearly typed
  - Already the default in the used programming language, Granule
  - Avoids problems; for example if a product (t_1, t_2) of two linear values would be unique by default, then it can be converted to an unrestricted value and can then be duplicated

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- They treat non-linearity and non-uniqueness as the same state: both are unrestricted
- Therefore, they have a single unrestricted modality $!$
- Progression: unique $\rightarrow$ unrestricted $\rightarrow$ linear

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- Typing contexts

$$\Gamma ::= \emptyset \mid \Gamma, x : A \mid \Gamma, x : [A]$$

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- Typing judgements $\Gamma \vdash t : A$

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Definition 1

Context addition $\Gamma_1 + \Gamma_2$ is the union of two contexts as long as every variable x that occurs in both contexts is assigned the same non-linear type in both contexts ($x : [A]$).

Typing rules: λ-calculus

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- For variables, abstraction and application
- Note that for VAR, the rest of the typing context has to be non-linear while the variable itself is linear

Typing rules: non-linear modality

- Introduction rule / promotion

$$\frac{[\Gamma] \vdash t : A}{[\Gamma] \vdash !t : !A} \, !_I$$

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- Elimination rule

$$\frac{\Gamma_1 \vdash t_1 : !A \qquad \Gamma_2, x : [A] \vdash t_2 : B}{\Gamma_1 + \Gamma_2 \vdash \text{let } !x = t_1 \text{ in } t_2 : B}\ !_E$$

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- Dereliction rule: non-linear variables can be used linearly

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma, x : [A] \vdash t : B}\ \mathrm{DER}$$

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$$\frac{\Gamma_1 \vdash t_1 : !A \qquad \Gamma_2, x : [A] \vdash t_2 : B}{\Gamma_1 + \Gamma_2 \vdash \text{let } !x = t_1 \text{ in } t_2 : B}\ !_E$$

- Dereliction rule: non-linear variables can be used linearly

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma, x : [A] \vdash t : B}\ \text{DER} \qquad\qquad \frac{\Gamma, P \vdash Q}{\Gamma, !P \vdash Q}\ !_L$$

Typing rules: uniqueness modality

- Borrowing: "forget" uniqueness guarantee

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- Necessitation: values without dependencies can be assumed unique

$$\frac{\emptyset \vdash t : A}{[\Gamma] \vdash *t : *A}\ \text{NEC}$$

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- Then we can compare performance of a language with both unique and linear types
- However, then we first need to define operational semantics

Heap model

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- Also include arrays and operations on them
  - As a use case for uniqueness

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- Uniqueness theorem is incorrect!

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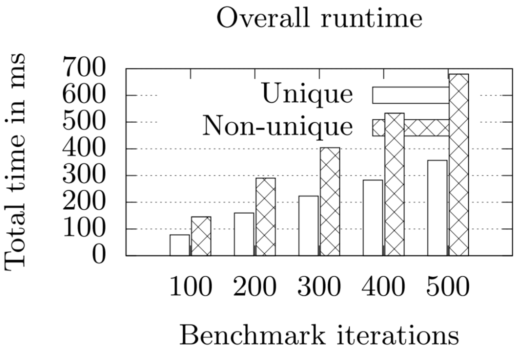
- Write operation updates unique array destructively in place
  - Allowed because we have uniqueness guarantee
- Compared performance with/without unique arrays

Performance

- Benchmark iteration: allocate list of 1000 arrays, populate arrays with values, traverse list to sum them up



Questions?

Counterexample uniqueness theorem

- Uniqueness: if a term reduces to a unique value, then any unique array references from the incoming heap or *new array references* that occur in that value are still unique
- Consider following heap + (well-typed) term

$$\emptyset \vdash *(\text{let } !x = \&(\text{newArray } 5) \text{ in } x) : *(\text{Array } A)$$

- Reduces to

$$a \mapsto_{\omega} \mathbf{arr}, x \mapsto_{\omega} a \vdash *a$$

- ω means that array reference a is not unique

Mistake in proof

- Proof of uniqueness theorem is by induction over typing rules
- Problem is with necessitation rule

$$\frac{\emptyset \vdash t : A}{[\Gamma] \vdash *t : *A}\ \textsc{nec}$$

- IH: "For a well-typed term $\Gamma \vdash t_1 : *B \ldots$"
- In the proof, they apply results of IH for $\emptyset \vdash t : A$ from $\textsc{nec}$
- A is not necessarily a unique type $*B$ for some B