MFoCS Seminar

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Today

The Titles of Two Papers

For MFoCS seminar, I offer the following papers:

1. "Type Theory with Explicit Universe Polymorphism" by Bezem, Coquand, Dybjer, Escardó
2. "Constructive set theory" by Myhill

Type Theory with Explicit Universe Polymorphism

Short Summary:

- This paper is related to **type theory** (see the course **Type Theory and Coq**)
- In type theory, we want **universe types**: this allows us to quantify over types
- For instance: prove that for all types A we have a function $A \to A$

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- Incorrect approach: add a type U that contains all types
- By Girard's paradox: this is inconsistent: $\forall (A : U), A$
- So we add many universes: $U_0 : U_1 : U_2 : U_3 : \ldots$

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- So we add many universes: $U_0 : U_1 : U_2 : U_3 : \dots$
- But: how do we prove statements for every universe? We need **universe polymorphism**
- This paper discusses several ways to add universe polymorphism to type theory

Constructive set theory

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- This paper: **constructive set theory** (CST aka CZF)
- Goal: **encode "Foundations of constructive analysis" by Bishop**
- In this book, a large amount of analysis got developed in constructive foundations
- This paper introduces the axioms of CZF (more subtle than just taking ZF and removing LEM)
- It also studies the **existence property** for a weaker form of CZF: if you can prove $\exists(a \in A), \varphi(a)$, then we can construct such an a s.t. $\varphi(a)$

Bibliography

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