MFoCS Seminar
“A Completeness Theorem for Probabilistic Regular Expressions”
Różowski, Silva
LICS 2024

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Regular expressions

Regular expressions $e, f \in \mathrm{RE}$ are a way to mathematically capture programs using

- actions $a, b, c, \ldots \in \Sigma$
- sequential composition $e \cdot f$
- non-deterministic choice $e + f$
- looping e^*

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Example

Let $e = (a + b) \cdot c \cdot (a + b)$, then $[\![e]\!] = \{aca, acb, bca, bcb\}$

Axioms for regular expressions

Theorem (Soundness & Completeness)

There exists a set of axioms A such that $e =_A f \iff [\![e]\!] = [\![f]\!]$.

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Examples of axioms

$$e + e = e$$
$$(e \cdot f) \cdot g = e \cdot (f \cdot g)$$
$$e \cdot (f + g) = e \cdot f + e \cdot g$$

Probabilistic regular expressions

Probabilistic regular expressions $e, f \in \mathrm{PRE}$ are a way to mathematically capture **probabilistc** programs using

- actions $a, b, c, \ldots \in \Sigma$
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- **probabilistic** choice $e +_{\mathbf{p}} f$
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Example

$e = (a +_{\frac{1}{2}} b) \cdot c \cdot (a +_{\frac{1}{2}} b)$, then $[\![e]\!] = \{(aca, \frac{1}{4}), (acb, \frac{1}{4}), (bca, \frac{1}{4}), (bcb, \frac{1}{4})\}$

Whats in the paper?

A Completeness Theorem for Probabilistic Regular Expressions - Różowski, Silva

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- Probabilistic regular expressions (PRE),
- Probabilistic regular languages ($\Sigma^* \to [0, 1]$),
- Probabilistic automata/transition systems,
- Axiomatisation of PRE,
- Proof of Soundness and Completeness using Coalgebra.