MFoCS Seminar

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Today

Constructive Logic

- Constructive logic is a language for constructions
- From a proof of $\exists (x \in X), P(x)$, one can "effectively" obtain such an x
- Basically, remove the law of exclude middle: $A \vee \neg A$

Realities for Constructive Mathematics: Realizability

We can view realizability as follows:

- We start with an underlying notion of computation (**partial combinatory algebra**)
- which we use to give an interpretation of **constructive** higher-order logic (**realizability interpretation**).
- Using general techniques, we obtain **realizability models** of type theory/set theory whose logic is determined by the realizability interpretation.

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I selected two papers for MFoCS seminar, and they are on the topic of realizability.

Paper I: Classical Realizability

Realizability is nice, but what if... yes or no yes

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Title: **On Krivine's realizability interpretation of classical second-order arithmetic** by Oliva and Streicher

- Classical realizability/Krivine's realizability: provide realizability models of **classical logic**
- Specifically, second-order arithmetic where one can quantify over all formulas
- Krivine's idea: λ-calculus with **call-with-current-continuation** (like Scheme)

This paper describes Krivine's realizability in 2 steps.

- $\neg\neg$-translation
- Constructive realizability

Paper II: Effectful Realizability

Realizability is nice, but what if... life has effect

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- In usual realizability models, one works with PCA. So: **determistic** computations that may be partial
- We can also consider realizablity models based on: **non-deterministic** computations, **stateful** computations, . . .

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- In usual realizability models, one works with PCA. So: **determistic** computations that may be partial
- We can also consider realizablity models based on: **non-deterministic** computations, **stateful** computations, ...

This paper:

- Defines the notion of **evidenced frame**
- Constructs realizability models using evidenced frames
- Instantiate it to effectful computations