State identification using characterizing sets or separating sequences

Kati Overbeeke

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Overview

- Motivation for testing
- Prerequisites
- Difference in the papers
- Minimal Separating Sequences for All Pairs of States
- Efficient State Identification for Finite State Machine-based Testing
- Comparison
- Conclusion

Motivation for testing

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  - Manual configuration
  - Test case design

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- Using a Finite State Machine
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- Other methods:
  - Property based testing
  - Extended FSM Models
  - Formal verification

Overview prerequisites

- Finite State Machines
- Separating sequence
- Characterisation set
- Test suite

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- Set of states, S
- Inputs, I
- Outputs, O
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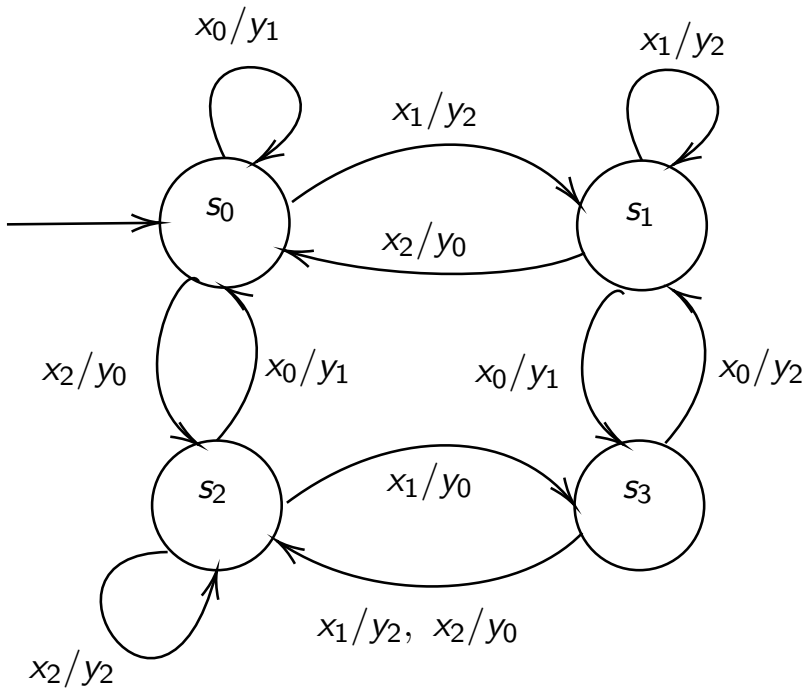

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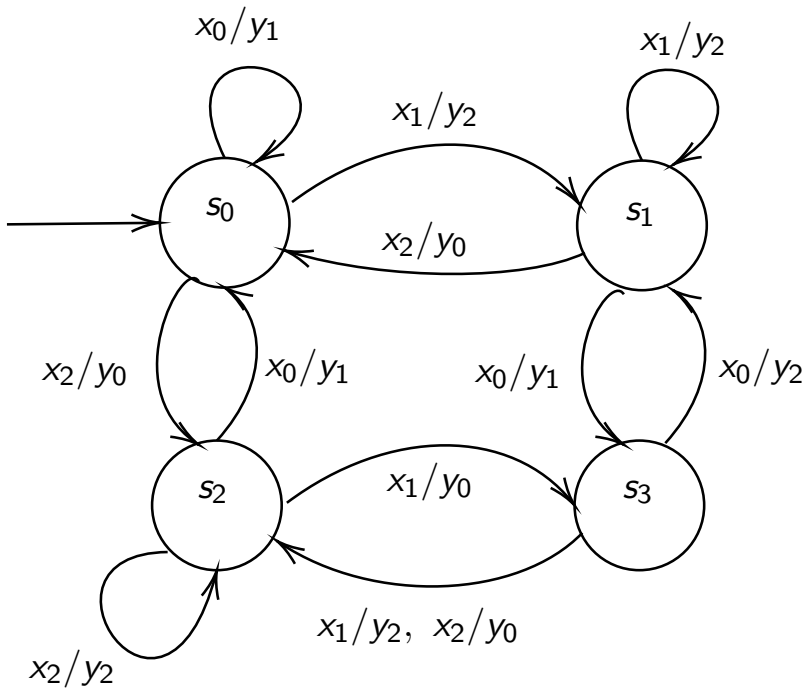

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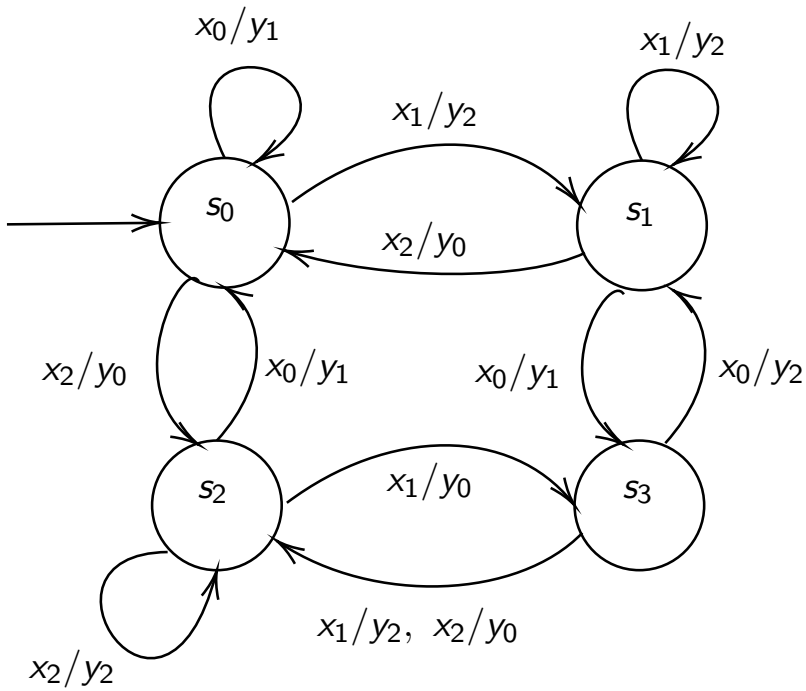

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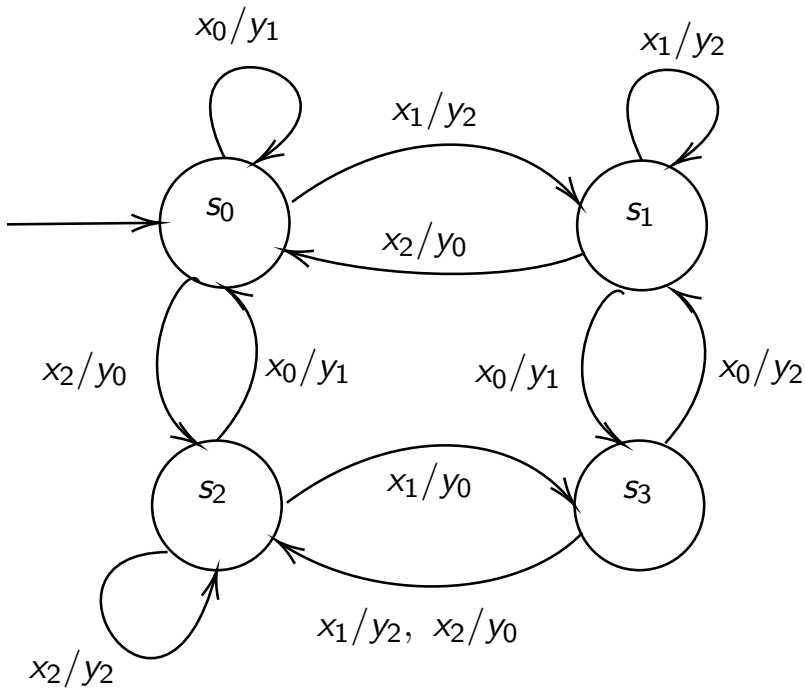

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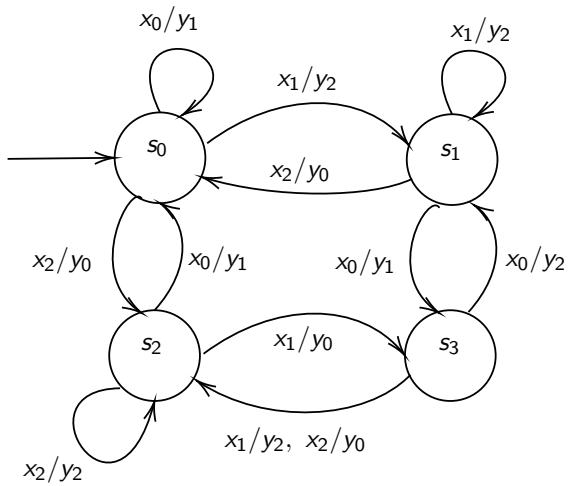

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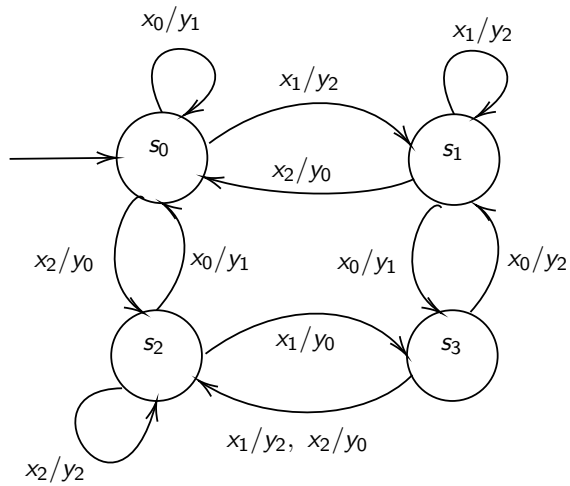

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x_0, x_1, x_2 do not separate s_0 and s_1.

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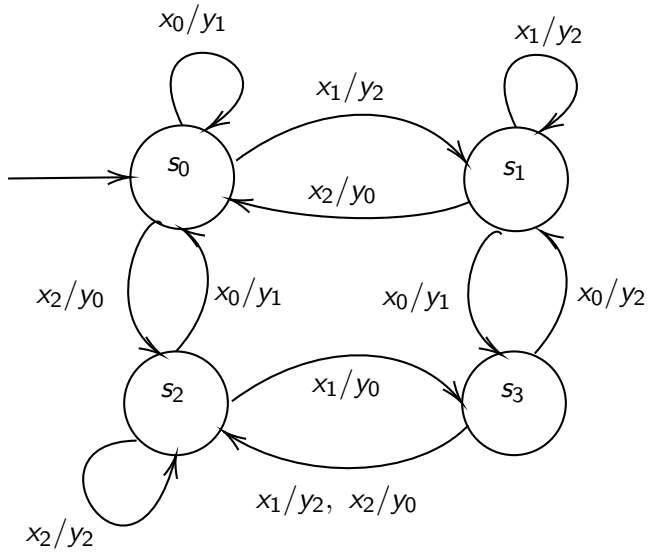

The set $W = \{x_0, x_2x_2\}$ is a characterizing set.

Test suite

- Set of test cases

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  - Input sequence
  - Corresponding expected output
- One of the definitions of the cost of a test suite:
  - Size of inputs and input sequences
  - Time required

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Minimal Separating Sequences for All Pairs of States

Minimal Separating Sequences for All Pairs of States

- Splitting Tree
- Splitting Tree to characterising set
- How to get a minimal splitting tree

Splitting tree

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For a Finite State Machine M, we define the splitting tree T with a finite set of nodes such that:

- a node u in T is labelled by a subset of S, denoted $l(u)$
- the root is labelled by S
- For each inner node u, $l(u)$ is partitioned by the labels of its children
- inner node u is associated with a sequence $\sigma(u)$ separating states in the children of u.

Splitting tree

Original algorithm

- Two ways to split:
  - split-output
    - Node is split with $a \in I$, if $\lambda(s, a) \neq \lambda(t, a)$
  - split-state
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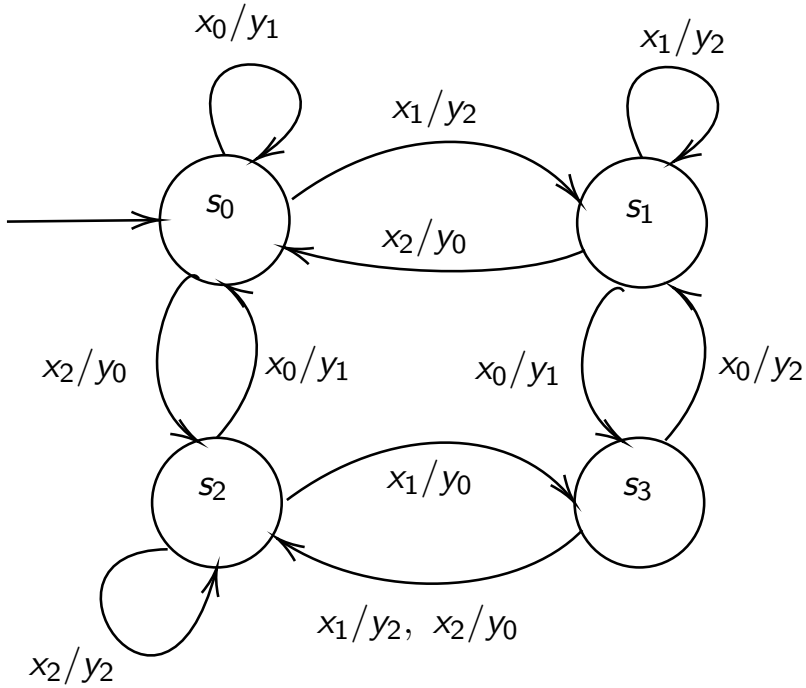

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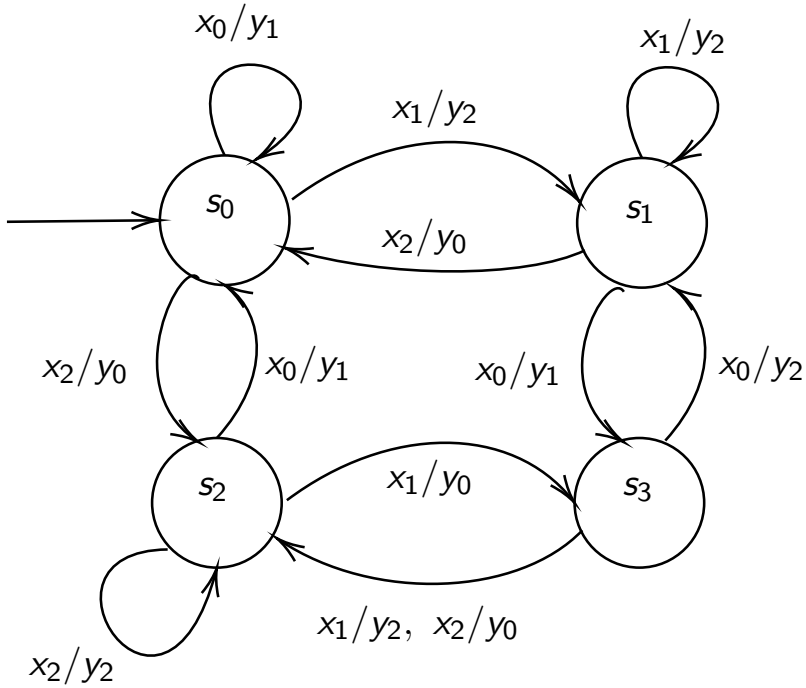

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$W = \{x_0, x_1, x_0x_0\}$

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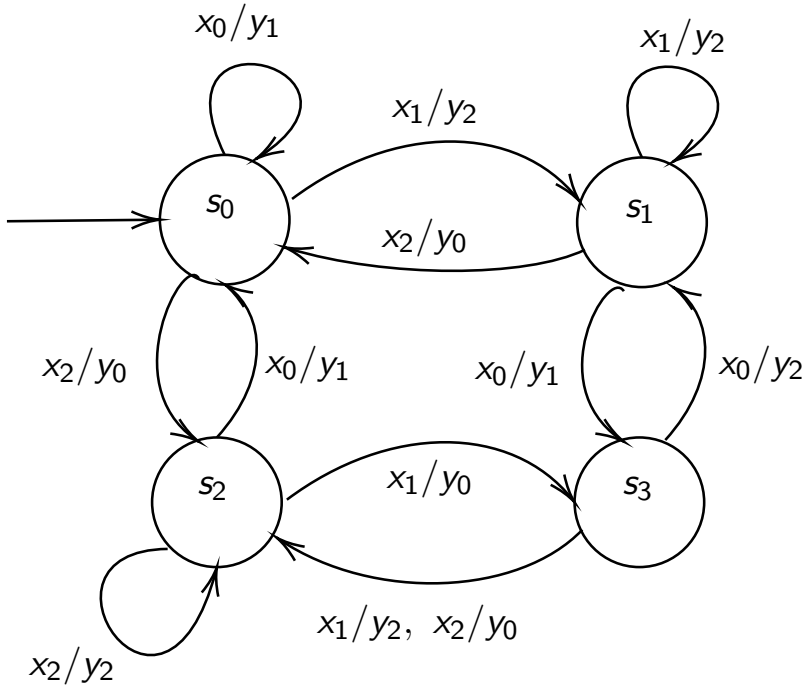

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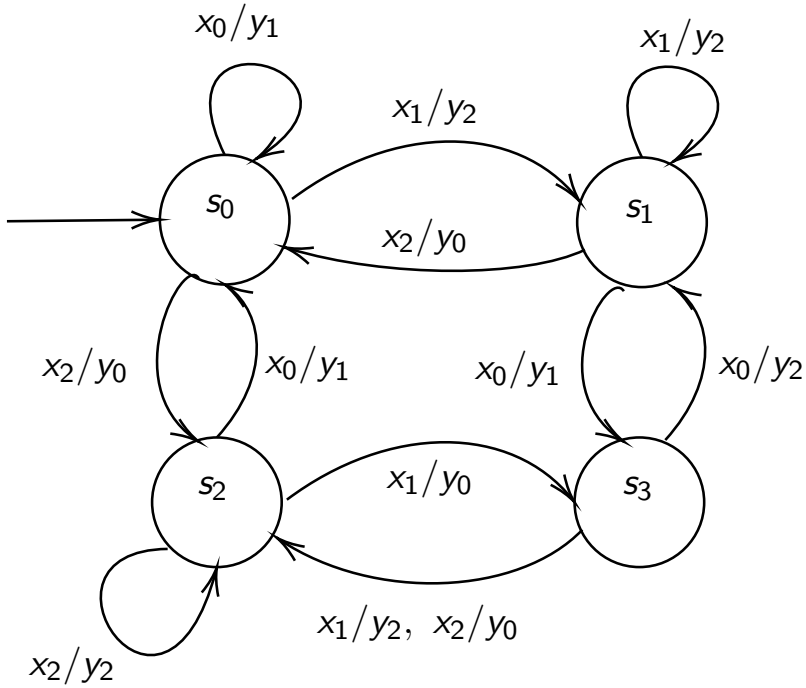

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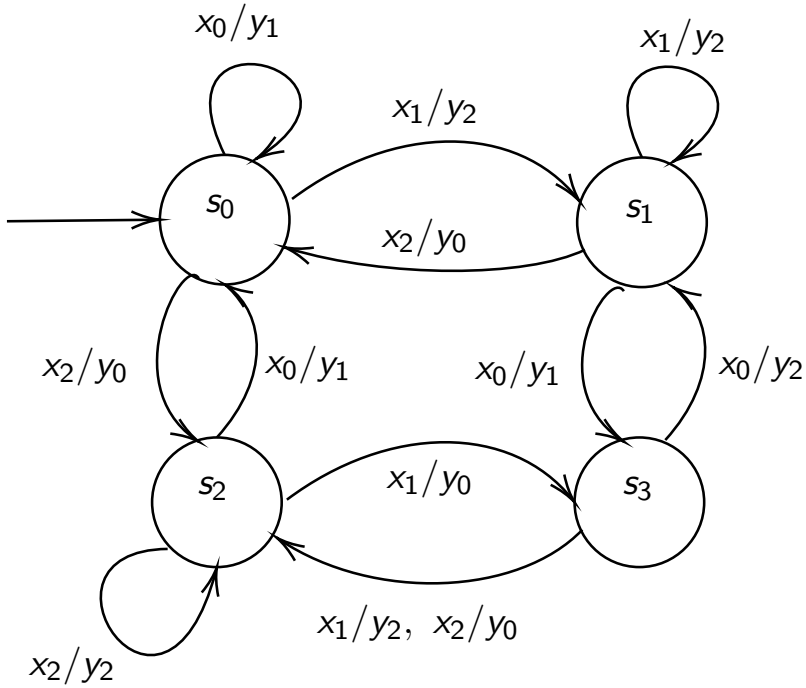

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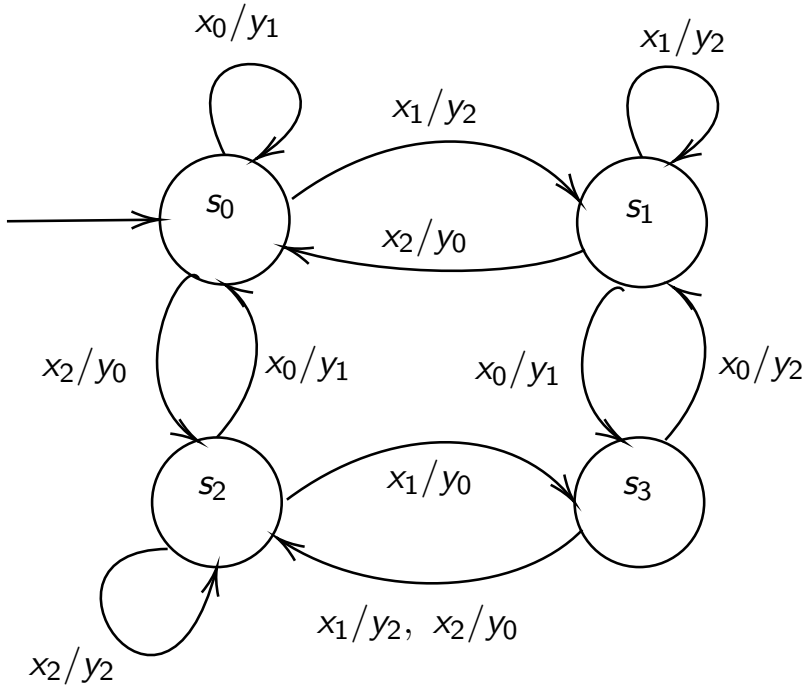

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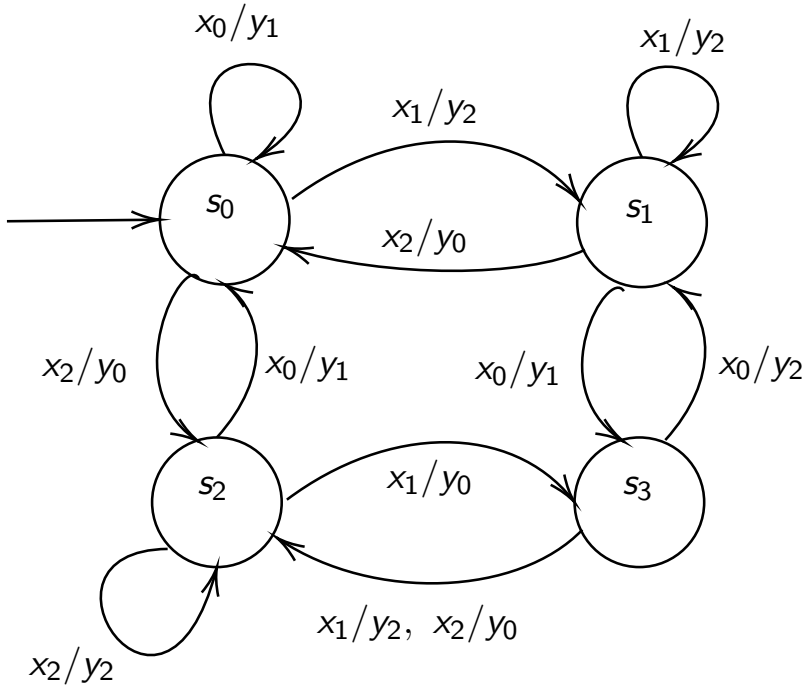

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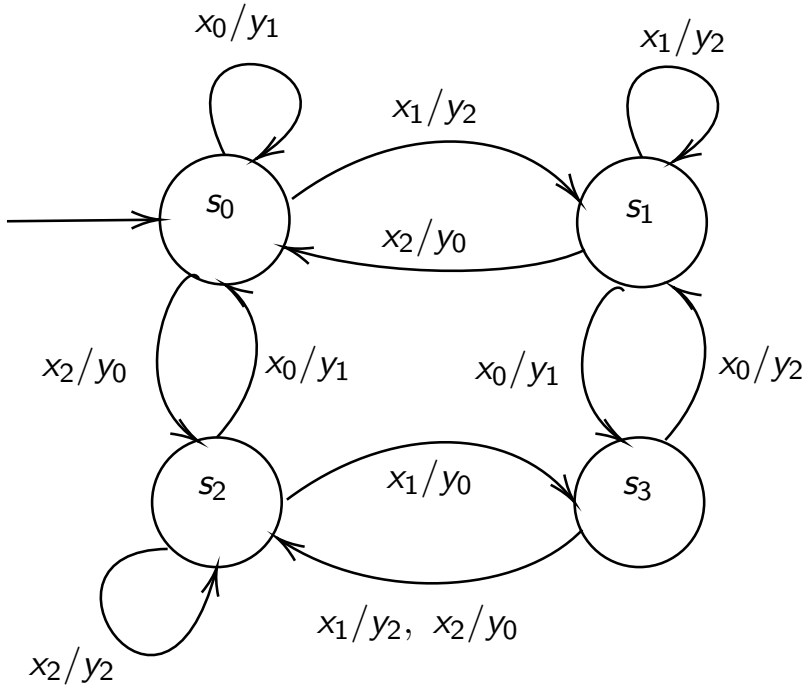

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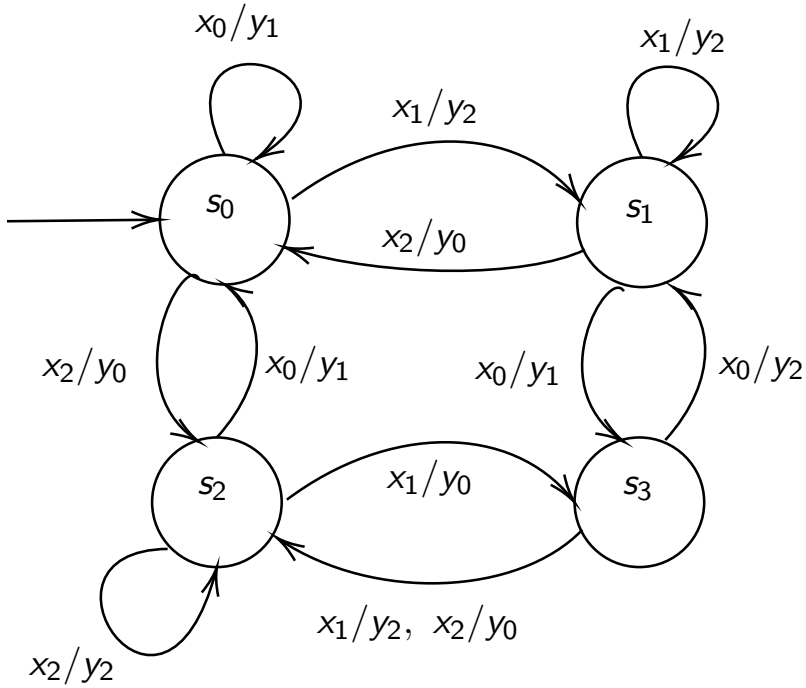

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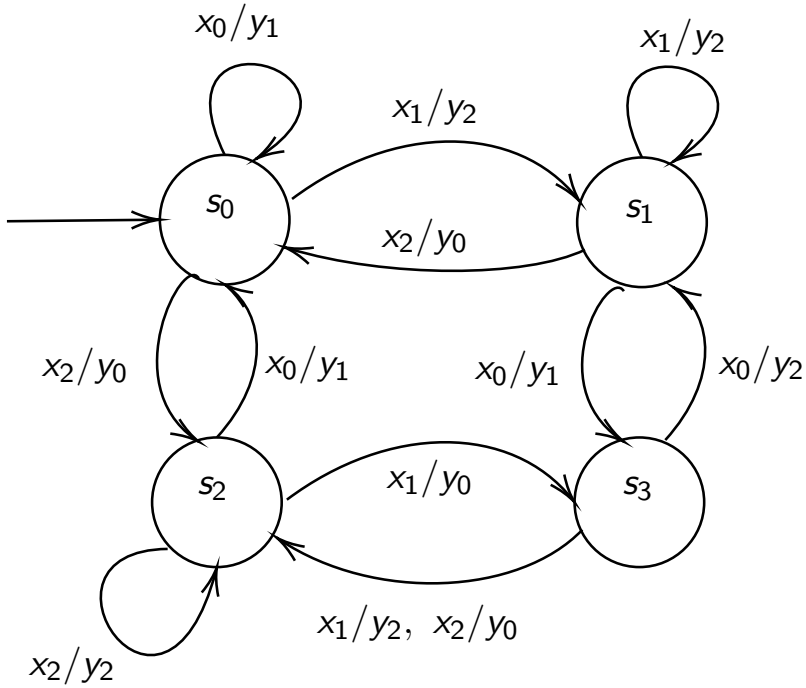

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Lowest Common Ancestor

Definition (Lowest Common Ancestor)

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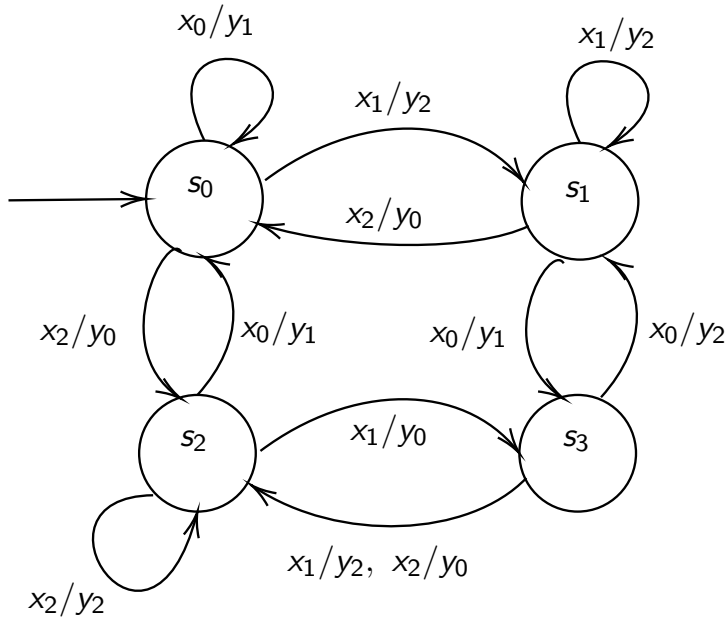

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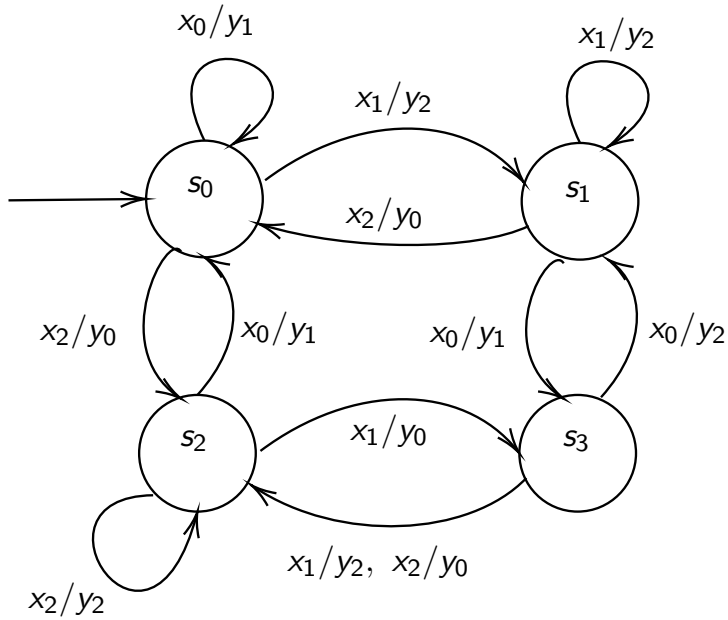

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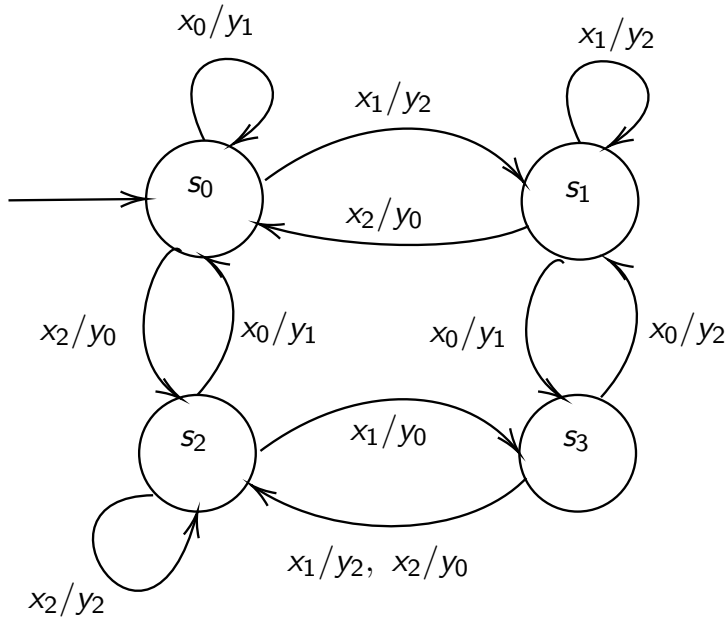

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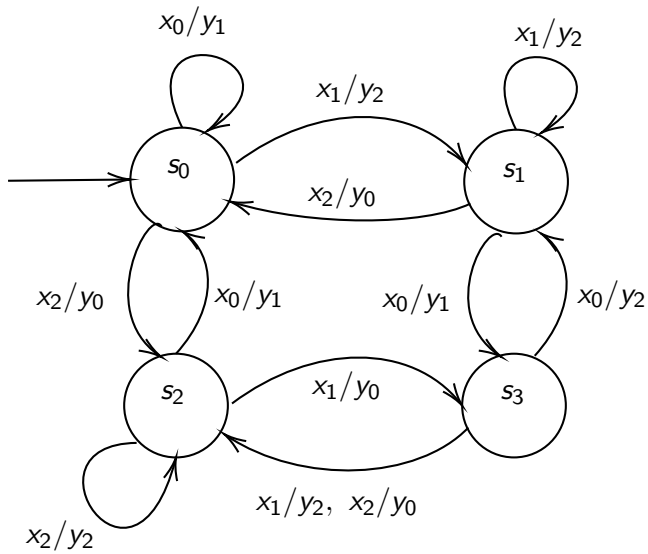

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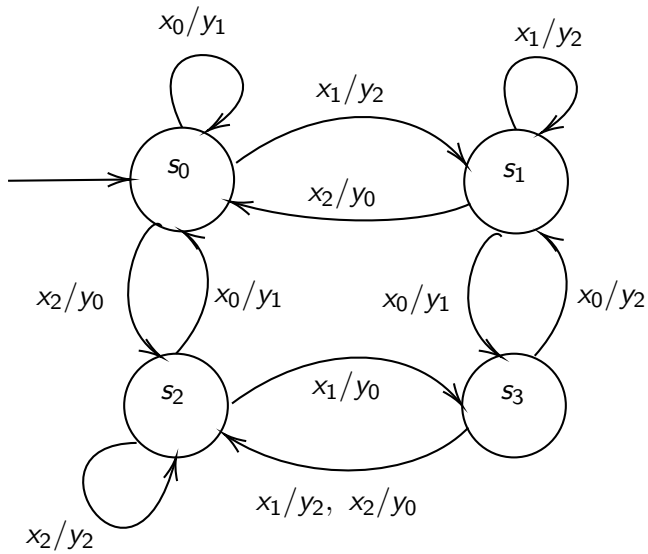

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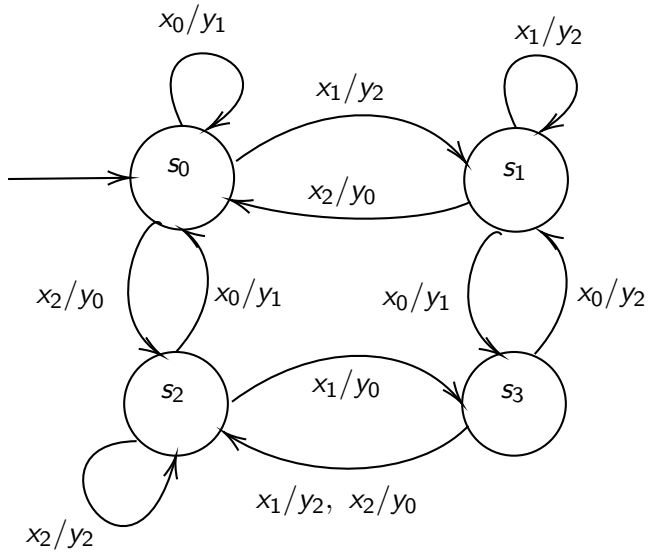

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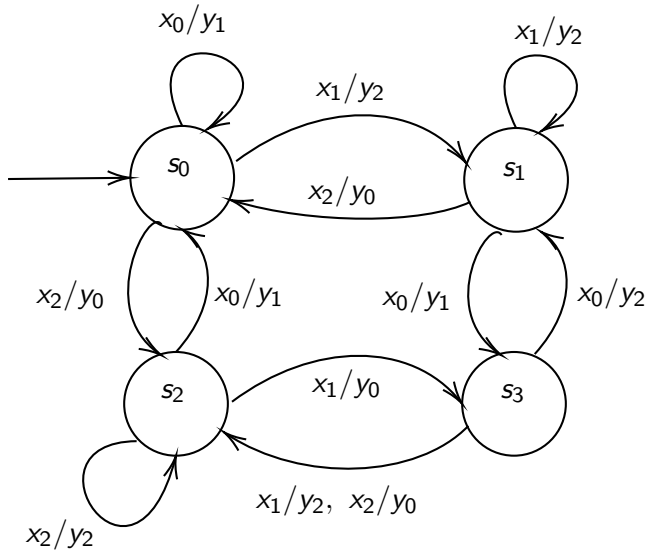

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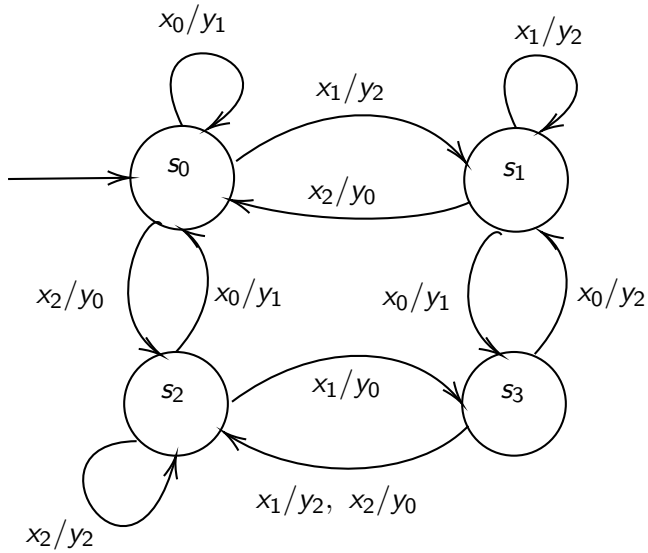

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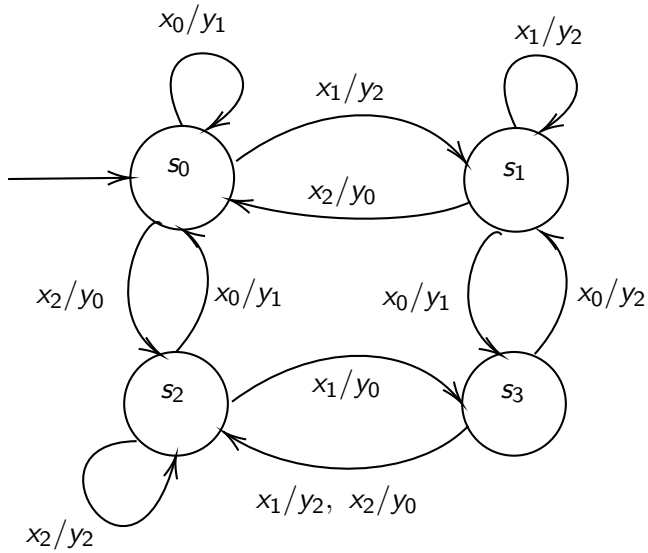

$[\{s_0, s_1\}, \{s_2\}, \{s_3\}]$ is 1-stable.
$[\{s_0, s_1\}, \{s_2\}, \{s_3\}]$ is not 2-stable.
$[\{s_0\}, \{s_1\}, \{s_2\}, \{s_3\}]$ is 2-stable.

Minimal splitting tree

Definition (minimal splitting tree)

T is minimal if for all states s, t in different leaves $\lambda(s, x) \neq \lambda(t, x)$ implies $|x| \geq |\sigma(lca(s, t))|$ for all $x \in I^*$.

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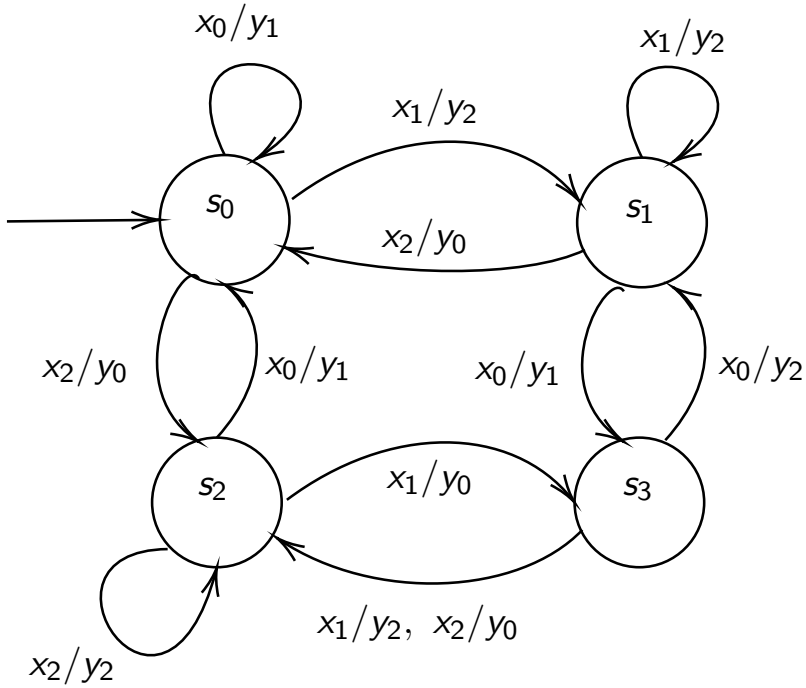

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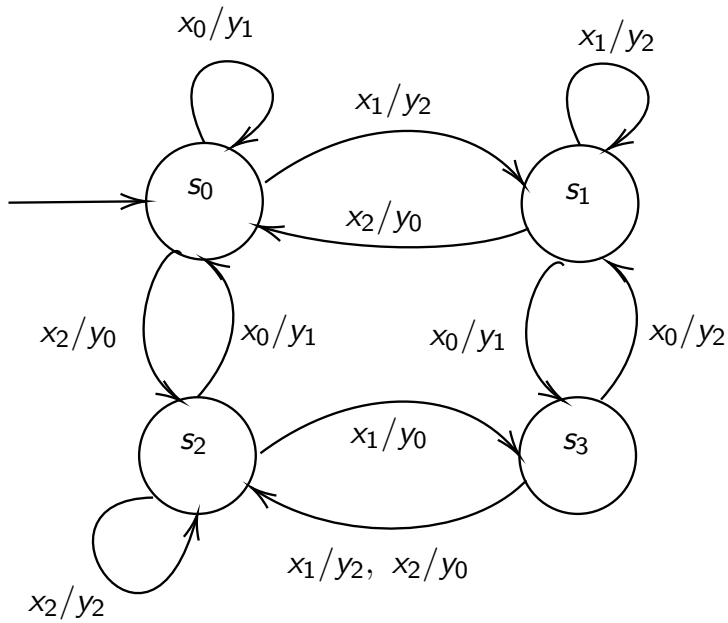

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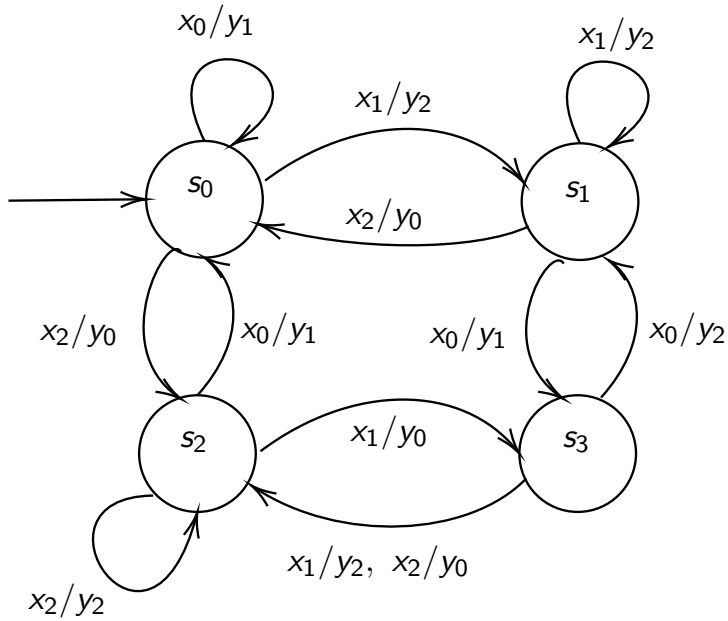

Note that this example is small, so it yields no difference.

Complexity of the algorithm

Speed up improvements:

Complexity of the algorithm

Speed up improvements:

- Building the split-state parts bottom up

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It was $\mathcal{O}(m\, n)$ in Moore's version.

Efficient state identification for finite state machine-based testing

Efficient state identification for finite state machine-based testing

- Redundant Characterizing set
- Identification path
- (B)O-Wset
- Complexity of the (B)O-Wset problem

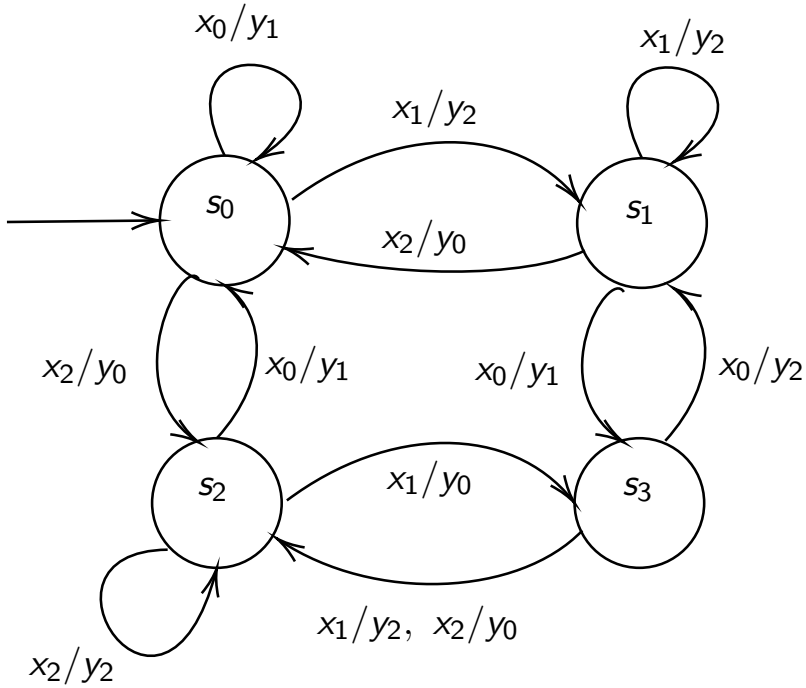
Redundant Characterizing set

Definition (Redundant Characterizing set)

A characterizing set W is redundant if either:

- $W \setminus \{w\}$ is a characterizing set for some $w \in W$.
- $W \cup \{w'\} \setminus \{w\}$ is a characterizing set for some $w \in W$ and a proper prefix w' of w.

Redundant Characterizing set Example



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$W_1 = \{x_0x_0x_0, x_2\}$ is redundant: x_0x_0 separates s_0, s_2 and s_1 and s_3.

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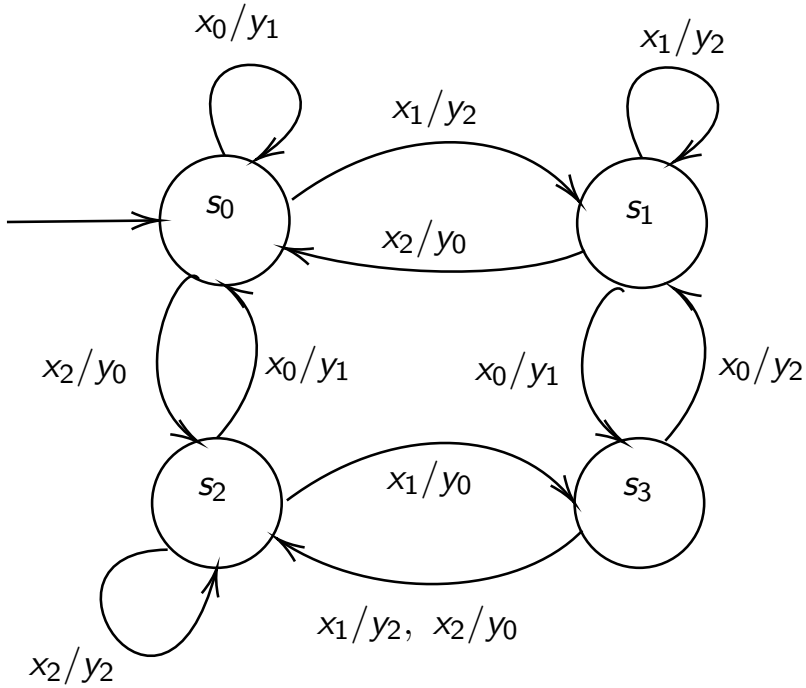

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$W_2 = \{x_2, x_0x_0\}$ is non-redundant.

Identification path

- Transfer-free identification path
- State identification path with transfers

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Example

An example of a transfer is when we have a sequence $x_1 b x_2$, and $b \notin W$.

Transfer-free identification path

Definition (Transfer-free identification path)

This is a path $\alpha_1\alpha_2 \ldots \alpha_k$ starting in the initial state, such that input$(\alpha_i) \in W$. Furthermore, for all $w \in W$, and for all states s, there is an α_i such that it corresponds with (s, w).

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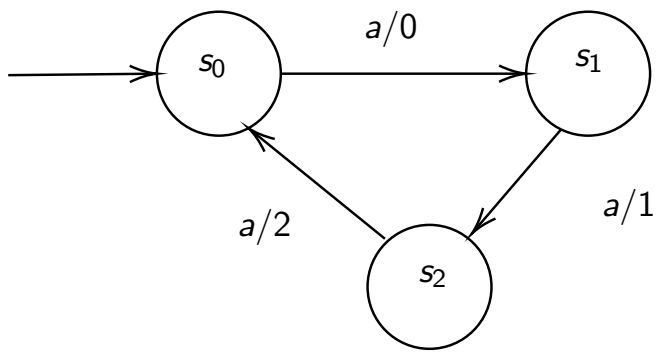

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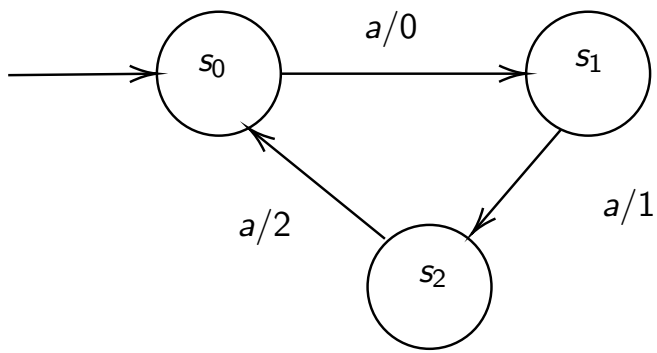

$W = \{a\}$ $s_0 \rightarrow_a s_1 \rightarrow_a s_2 \rightarrow_a s_0$

State identification path with transfers

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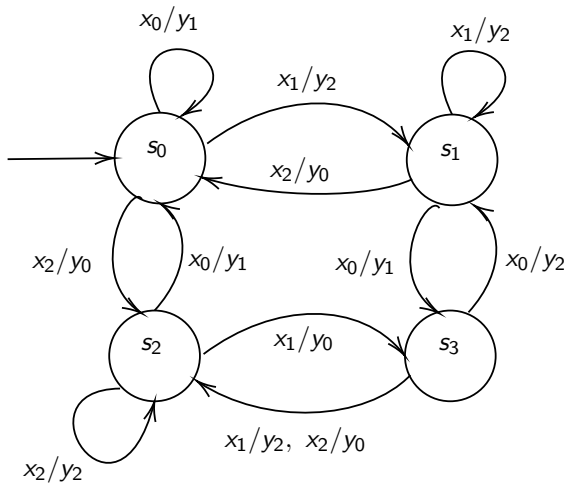

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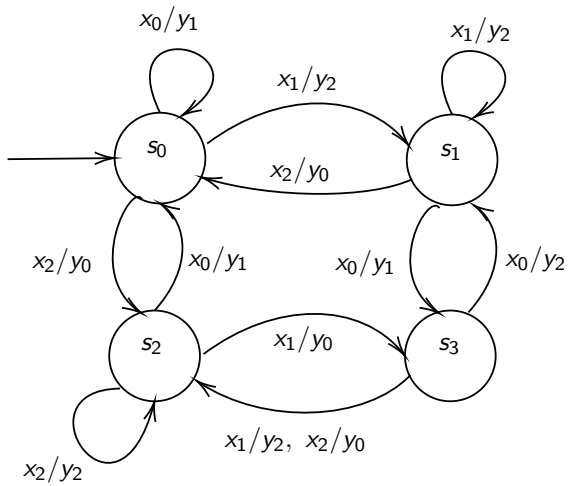


$W = \{x_1, x_2x_2\}$, $s_0 \rightarrow_{x_1} s_1 \rightarrow_{x_1} s_1 \rightarrow^*_{x_2x_2} s_2 \rightarrow_{x_1} s_3 \rightarrow_{x_1} s_2 \rightarrow^*_{x_2x_2}$
$s_2 \rightarrow_{x_0} s_0 \rightarrow^*_{x_2x_2} s_2 \rightarrow^*_{x_0x_1x_0} s_3 \rightarrow^*_{x_2x_2} s_2$

(B)O-Wset

- Minimal characterising set
- O-Wset
- BO-Wset

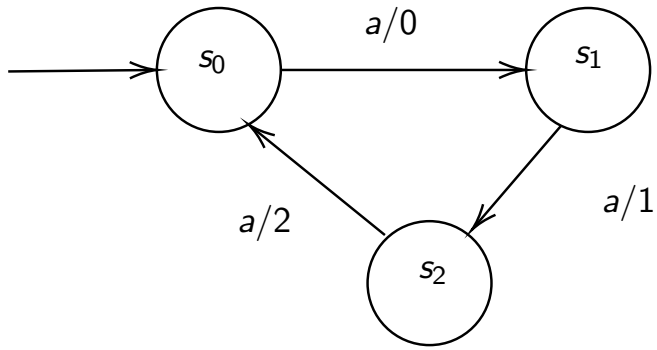
Minimal characterising set

Definition (Minimal characterising set)

A characterising set W is called minimal if for all $s, s' \in S$ where $s \neq s'$ there is a prefix w' of a sequence $w \in W$ such that:

- w' separates s and s'
- no shorter sequence separates s and s'

Minimal Characterizing set - Example 1



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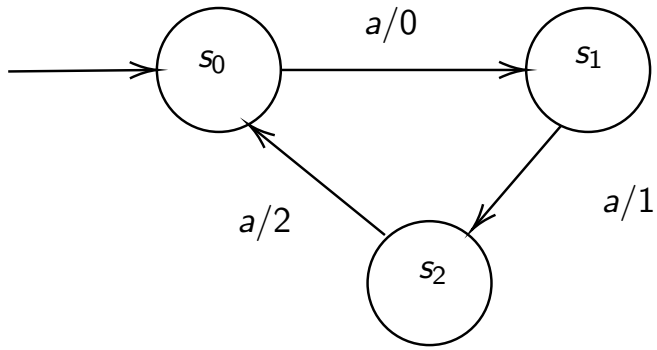

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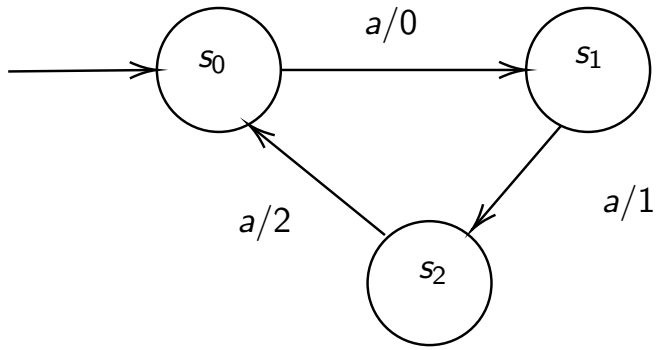

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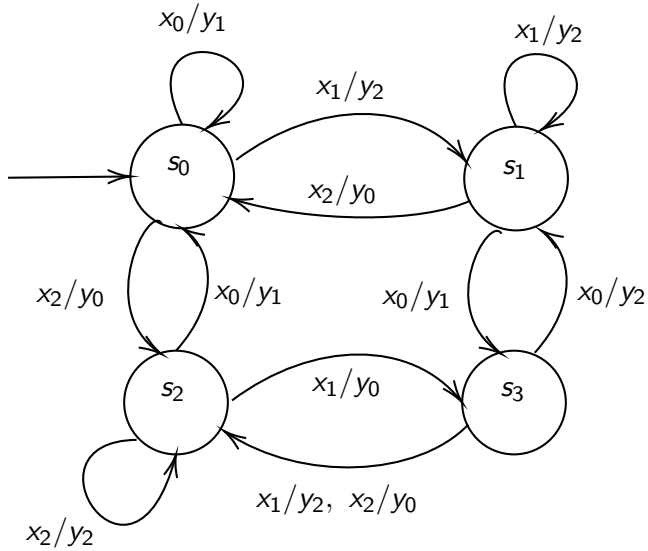
- a separates s_0, s_1, s_2
- No shorter sequence separates the states
- So $W = \{a\}$ is minimal

Minimal Characterizing set - Example 2



Example

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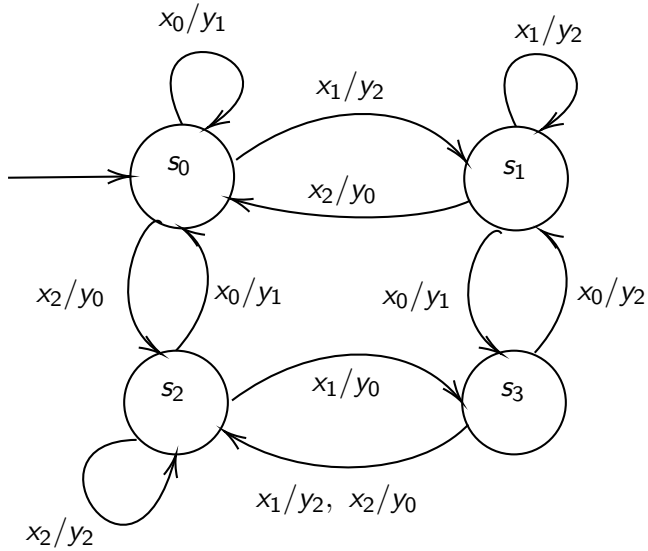

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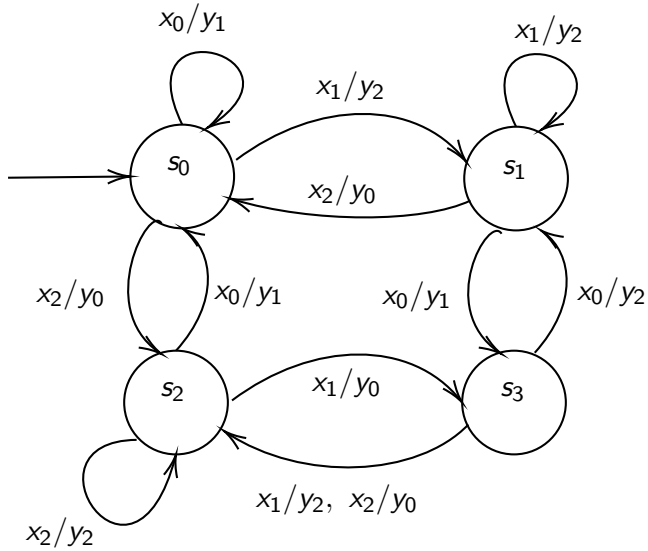

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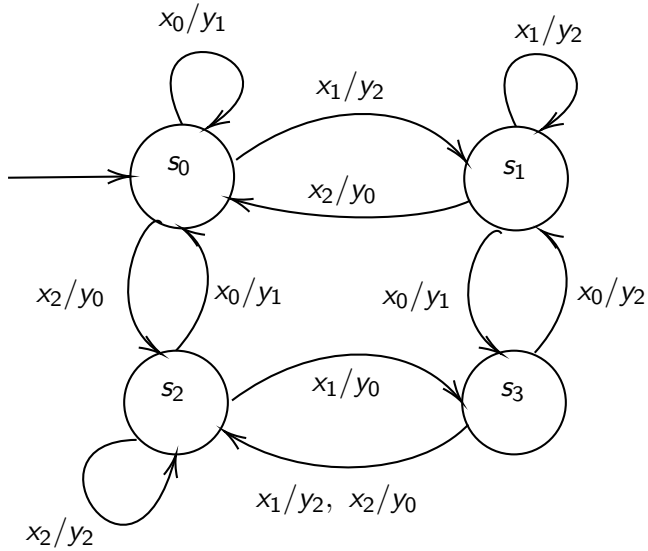

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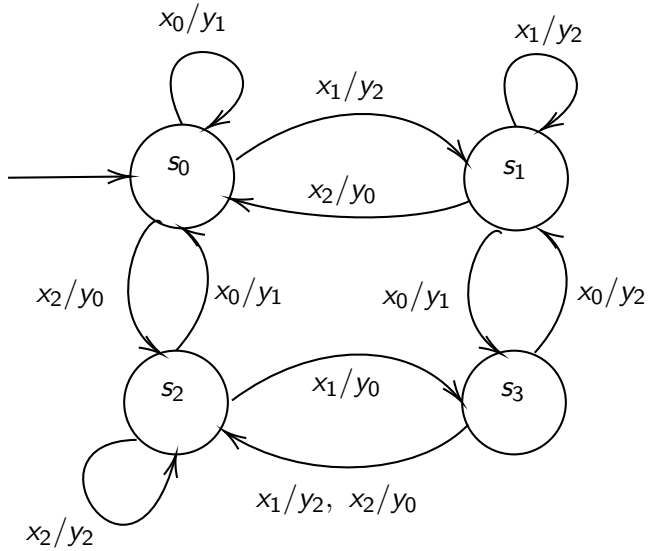

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- x_2x_2 separates s_0 and s_1 and no shorter sequence does.
- So $W = \{x_1, x_2x_2\}$ is minimal.

O-Wset

Definition

W is an O-Wset if

- W is a minimal characterizing set
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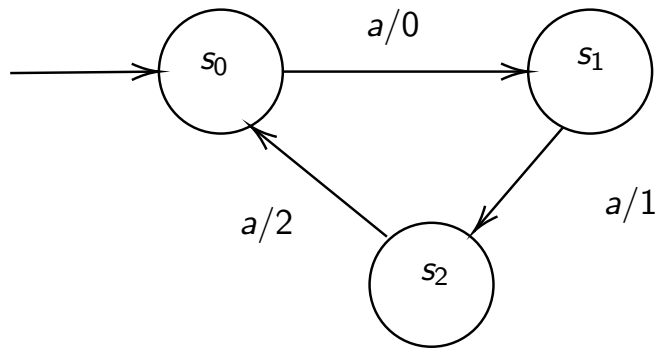

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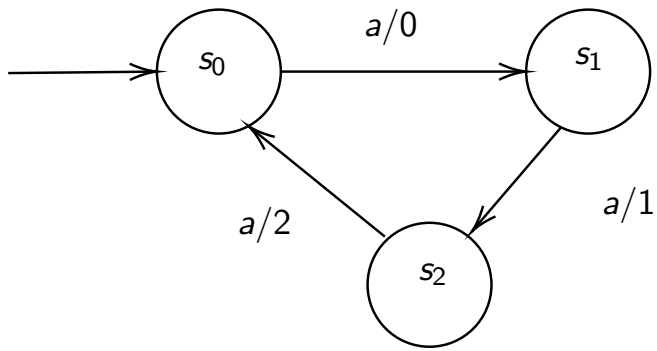

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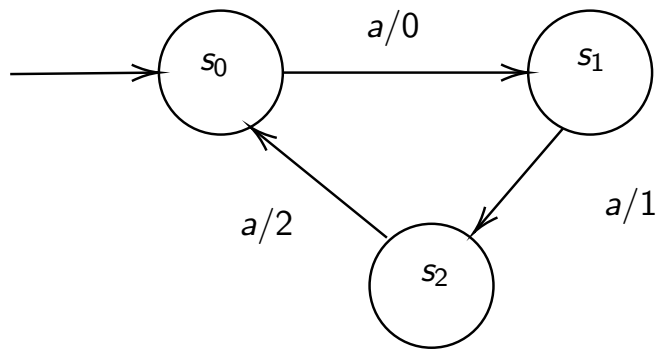

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BO-Wset

Definition

W is a BO-Wset if there exists a k such that:

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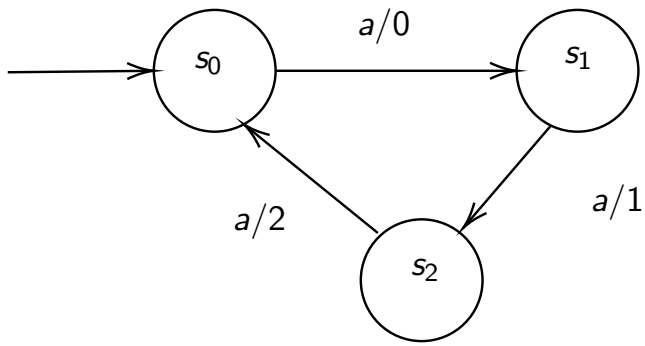
- There exists a state identification path with at most k transfers.

BO-Wset, $k = 0$



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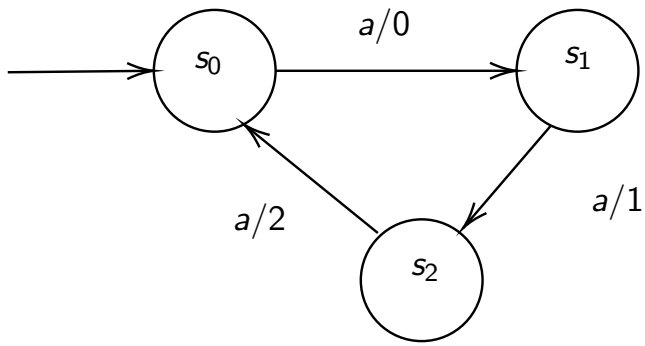

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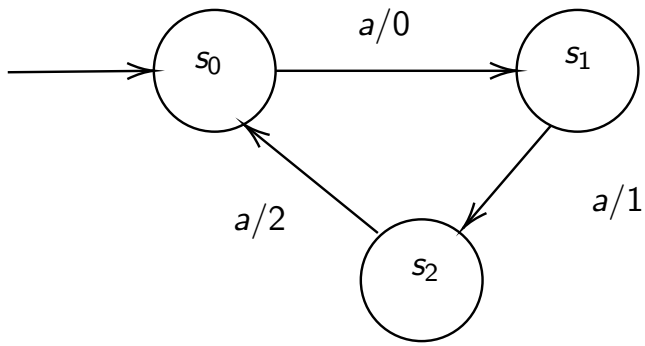

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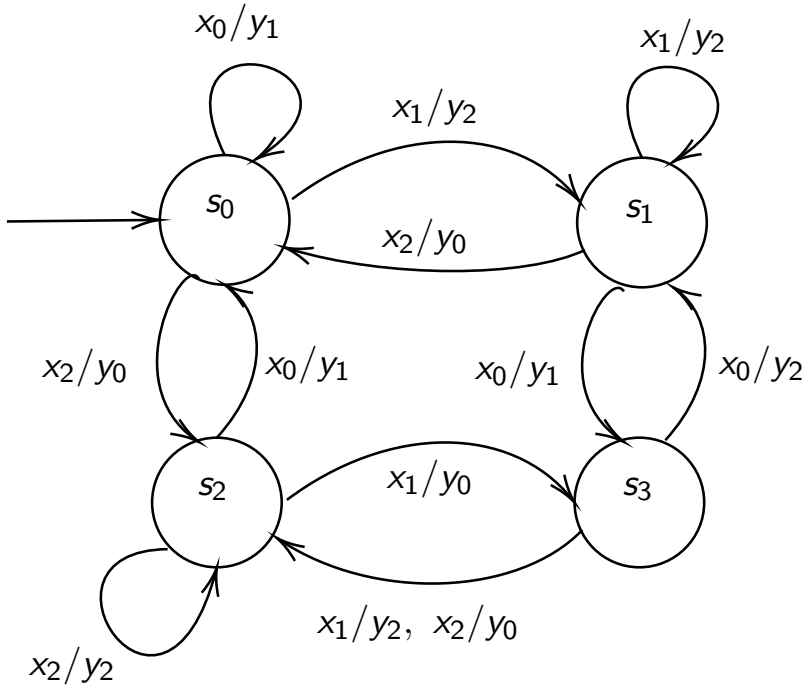
BO-Wset, $k = 4$



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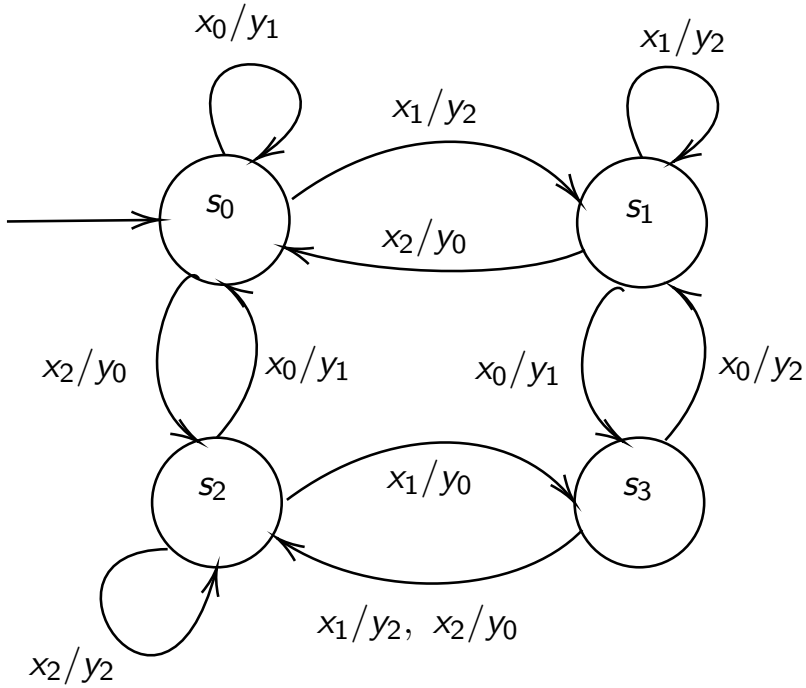
BO-Wset, $k = 4$



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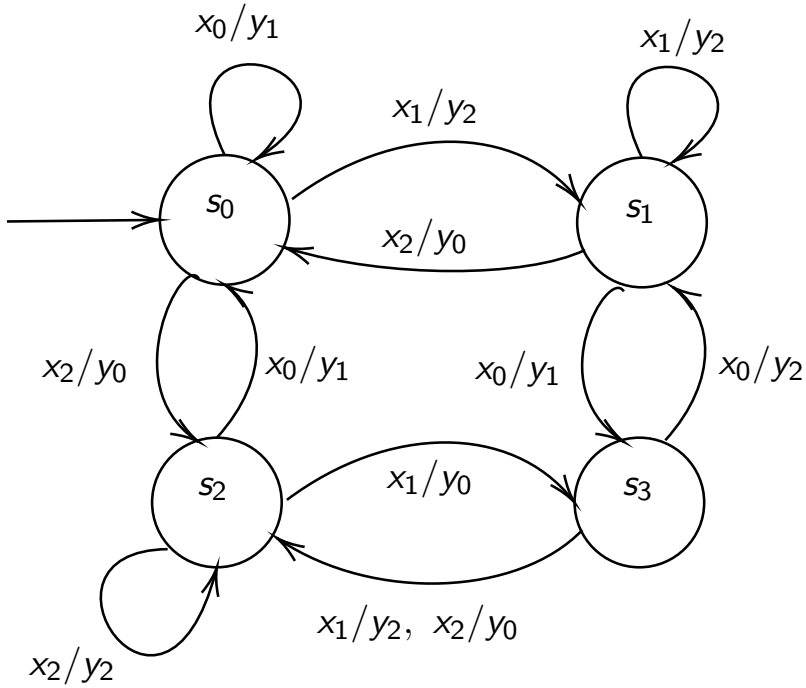

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$s_2 \to^*_{x_0x_1x_0} s_3 \to^*_{x_2x_2} s_2$ has at most $k = 4$ transfers.

Complexity of finding a (B)O-WSet

- NP-complete

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Example

$U := \{1, 2, 3, 4, 5, 6\}$, $C := \{\{1, 2, 3\}, \{2, 3, 4\}, \{4, 5, 6\}\}$.
Is there a satisfying assignment?

X3C

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Is there a satisfying assignment?

X3C

- Set of elements U
- Finite set of subsets of the elements (of size 3): C
- Find $\mathbb{C} \subseteq C$
  - $\bigcup_{e_i \in \mathbb{C}} e_i = U$
  - for $e_i, e_j \in \mathbb{C}$ with $i \neq j$: $e_i \cap e_j = \emptyset$
- Complexity: NP-complete

Example

$U := \{1, 2, 3, 4, 5, 6\}$, $C := \{\{1, 2, 3\}, \{2, 3, 4\}, \{4, 5, 6\}\}$.
Is there a satisfying assignment?
Yes $\mathbb{C} = \{\{1, 2, 3\}, \{4, 5, 6\}\}$

Example

$U := \{1, 2, 3, 4, 5, 6\}$, $C := \{\{1, 3, 5\}, \{2, 3, 4\}, \{4, 5, 6\}\}$.
Is there a satisfying assignment?
No.

X3C to O-Wset

Theorem

There is a satisfying assignment $\mathbb{C}$ for a given X3C problem iff there is a transfer-free path in the FSM M.

X3C to O-Wset

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There is a satisfying assignment $\mathbb{C}$ *for a given X3C problem iff there is a transfer-free path in the FSM M. M is defined by the mapping* $S := \{s_i | u_i \in U\} \cup \{s_{|U|+1}\}$, $I := \{x_i | c_i \in C\}$,

$O := \{y_i | u_i \in U\} \cup \{0\}$,

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$$\delta(s_i, x_j) = \begin{cases} s_{i+1} \text{ if } u_i \in e_j \\ s_i \text{ otherwise} \end{cases}.$$

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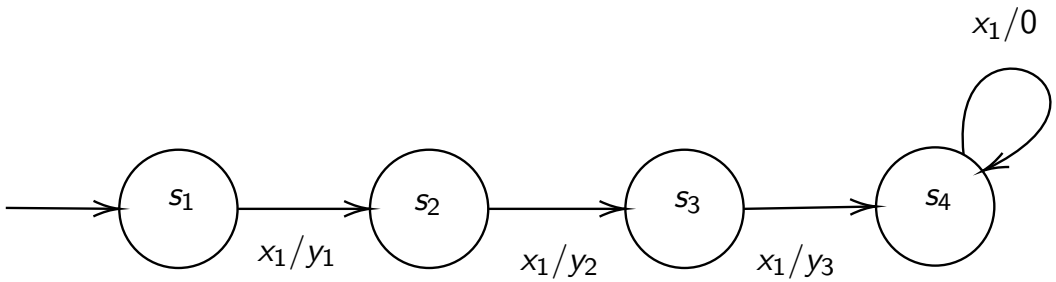
$\delta(s_i, x_j) = \begin{cases} s_{i+1} \text{ if } u_i \in e_j \\ s_i \text{ otherwise} \end{cases}$.

Example

$U = \{u_1, u_2, u_3\}$, $C = \{e_1\} = \{\{u_1, u_2, u_3\}\}$.

Mapping X3C to O-Wset

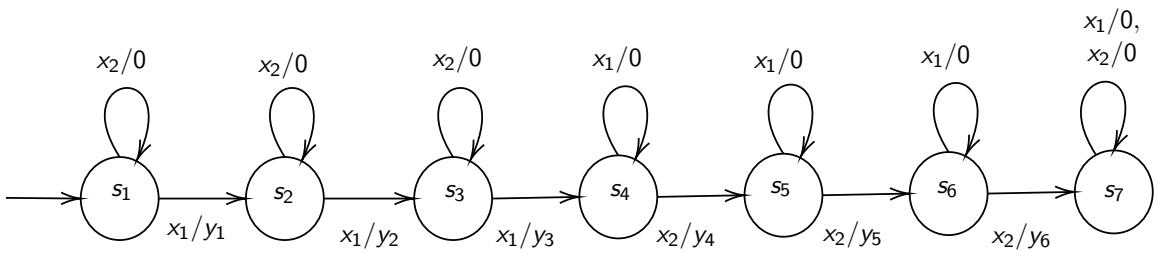
$U = \{u_1, u_2, u_3\}$, $S = \{s_1, s_2, s_3, s_4\}$, $C = \{e_1\} = \{\{u_1, u_2, u_3\}\}$,
$I = \{x_1\}$,
$O = \{y_1, y_2, y_3, 0\}$.



Transition for state s_1: For x_1, u_1 in e_1 => goes to s_2, with output y_1 as it is state 1.
Transition for state s_2: For x_1, u_1 in e_1 => goes to s_3,with output y_2 as it is state 2.
Transition for state s_3: For x_1, u_1 in e_1 => goes to s_4,with output y_3 as it is state 3.
Transition for state s_4: For x_1, u_1 not in e_1 => goes to s_4, with output 0.

X3C to O-Wset - path

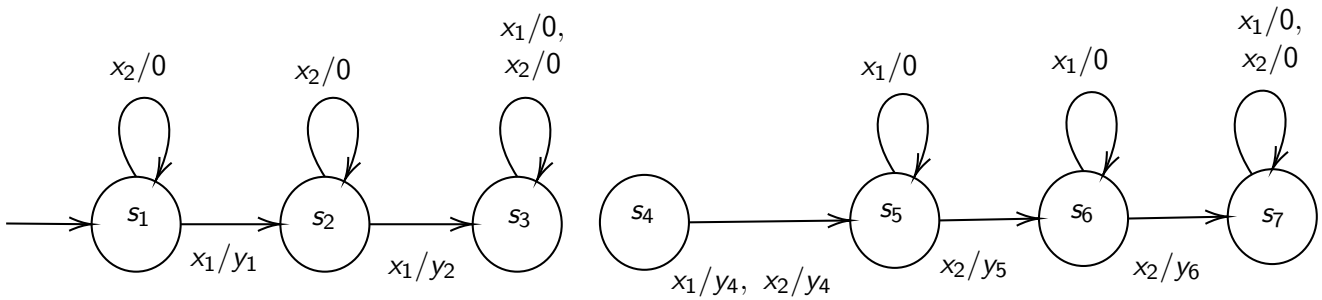
$U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$,
$C = \{e_1, e_2\} = \{\{u_1, u_2, u_3\}, \{u_4, u_5, u_6\}\}$,
$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$,
$I = \{x_1, x_2\}$, $O = \{y_1, y_2, y_3, y_4, y_5, y_6, 0\}$



$W := \{x_1, x_2\}$.
Path is : $s_1 \rightarrow_{x_2} s_1 \rightarrow_{x_1} s_2 \rightarrow_{x_2} s_2 \rightarrow_{x_1} s_3 \rightarrow_{x_2} s_3 \rightarrow_{x_1} s_4 \rightarrow_{x_1}$
$s_4 \rightarrow_{x_2} s_5 \rightarrow_{x_1} s_5 \rightarrow_{x_2} s_6 \rightarrow_{x_1} s_6 \rightarrow_{x_2} s_7 \rightarrow_{x_1} s_7 \rightarrow_{x_2} s_7$

X3C to O-Wset - path

$U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$,
$C = \{e_1, e_2\} = \{\{u_1, u_2, u_4\}, \{u_4, u_5, u_6\}\}$,
$S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$,
$I = \{x_1, x_2\}$, $O = \{y_1, y_2, y_3, y_4, y_5, y_6, 0\}$



No path.

NP completeness for (B)O-Wset

- Mapping from X3C to O-Wset

NP completeness for (B)O-Wset

- Mapping from X3C to O-Wset
- X3C to BO-Wset

Comparison of the papers

Recap: Minimal splitting tree or (B)O-Wset?

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Is a combination possible?

Conclusion

- Splitting Tree
- Splitting Tree to characterising set
- (B)O-Wset
- Comparison of methods