

# Coalgebra: Lecture 1

Jurriaan Rot

Radboud University, 2018



# Introducing the lecturer(s)

- Jurriaan Rot — [jrot@cs.ru.nl](mailto:jrot@cs.ru.nl)  
<http://jurriaan.me>
- Also 2 lectures given by  
Aleks Kissinger — [aleks@cs.ru.nl](mailto:aleks@cs.ru.nl)



<https://www.cs.ru.nl/A.Kissinger/>

## Website

<http://www.cs.ru.nl/~jrot/coalg18/>

Course information (lecture dates, exercises, literature, announcements, . . . ) on the **website**

Please also register for the course (email announcements and grades via Brightspace)



# Time and date

## Lectures

- **Mondays** 13:30 - 15:30, MERC I 00.28
- Weeks 37-42 and 45-51 (14 lectures total)
- Schedule announced on the website

## Exercise class

- **Thursday** 13:30 - 15:30, MERC I 00.28
- **No class this Thursday (Sept 6)**
- Weeks 37-42 and 45-51
- Meant to **practice** and **ask questions**: no new material

## Literature

We'll use several sources, in particular research papers; announced on the website.

First couple lectures, a new (draft) book:

- Jan Rutten. The method of coalgebra: exercises in coinduction.  
<http://www.cwi.nl/~janr/papers/files-of-papers/course.pdf>

We will also sometimes use:

- Bart Jacobs. Introduction to coalgebra, version 2.0, 2012.  
<http://www.cs.ru.nl/B.Jacobs/CLG/JacobsCoalgebraIntro.pdf>  
Official version published by Cambridge University Press, 2016.

Second half (mostly): lecture notes

## Assessment

- Two graded assignments: end of October, end of December (separate from the homework in the exercise classes, which is not graded)
- A final exam (January 11th 2019)
- Final grade:  $(H+E)/2$ , where H is the grade given for the homework assignments and E is the grade given for the final exam

# Coalgebra

- Mathematical theory of **state-based systems**

# Coalgebra

- Mathematical theory of **state-based systems**
- Uniform study of many types of systems and models; various kinds of automata, (infinite) data structures such as sequences and trees, probabilistic systems, . . .

# Coalgebra

- Mathematical theory of **state-based systems**
- Uniform study of many types of systems and models; various kinds of automata, (infinite) data structures such as sequences and trees, probabilistic systems, . . .
- Parametric in the type of system, which gives, for each type, a notion of **behavioural equivalence** and **semantics**, and associated **proof methods**

# Coalgebra

- Mathematical theory of **state-based systems**
- Uniform study of many types of systems and models; various kinds of automata, (infinite) data structures such as sequences and trees, probabilistic systems, . . .
- Parametric in the type of system, which gives, for each type, a notion of **behavioural equivalence** and **semantics**, and associated **proof methods**
- Applications, fundamental insights, new perspectives

## A small bit of history

- Machines and data structures in category theory (70's, 80's)
- Behavioural equivalence of processes (Milner & Park, 80's)
- Processes and non-wellfounded set theory (Peter Aczel (late 80's))
- Universal coalgebra (Jan Rutten and others – from 90's on)
- By the end of the 90's: active and diverse area of research, conference & workshop of it's own

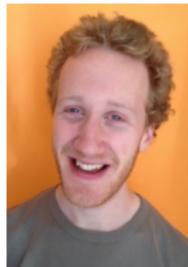


## Coalgebra research today

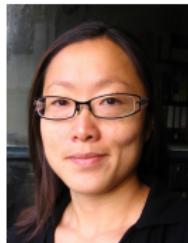
Active community studying the fundamental theory of coalgebras, as well as applications in/connections to:

- automata theory
- programming language semantics
- concurrency theory
- probabilistic systems
- modal logic
- theorem proving
- type theory
- ...

# Coalgebra in Nijmegen



and people who now work elsewhere:



## What will you learn

In this course:

- basic theory — modelling systems as coalgebras

# What will you learn

In this course:

- basic theory — modelling systems as coalgebras
- applying the coalgebraic notions of behavioural equivalence and coinduction

## What will you learn

In this course:

- basic theory — modelling systems as coalgebras
- applying the coalgebraic notions of behavioural equivalence and coinduction
- the interplay between algebra and coalgebra, induction and coinduction

## What will you learn

In this course:

- basic theory — modelling systems as coalgebras
- applying the coalgebraic notions of behavioural equivalence and coinduction
- the interplay between algebra and coalgebra, induction and coinduction
- using coalgebraic/coinductive proof techniques;
  - *highlight: a recent breakthrough algorithm for equivalence of non-deterministic automata*

# Overview

## Basic theory of coalgebras and coinduction (first quarter)

- streams
- categories, coalgebras, bisimulation
- automata

## Algebra and coalgebra (second quarter)

- algebras, induction
- monads
- distributive laws

## Bisimulation and coinduction: advanced topics (second quarter)

- lattices and fixed point theory
- minimization
- proof techniques and algorithms, applications