

# Coalgebra: Lecture 1

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Radboud University, 2018



## Introducing the lecturer(s)

- Jurriaan Rot — `jrot@cs.ru.nl`  
<http://jurriaan.me>
- Also 2 lectures given by  
Aleks Kissinger — `aleks@cs.ru.nl`



<https://www.cs.ru.nl/A.Kissinger/>



## Website

<http://www.cs.ru.nl/~jrot/coalg18/>

Course information (lecture dates, exercises, literature, announcements, ...) on the **website**

Please also register for the course (email announcements and grades via Brightspace)



## Time and date

### Lectures

- **Mondays** 13:30 - 15:30, MERC I 00.28
- Weeks 37-42 and 45-51 (14 lectures total)
- Schedule announced on the website

### Exercise class

- **Thursday** 13:30 - 15:30, MERC I 00.28
- **No class this Thursday (Sept 6)**
- Weeks 37-42 and 45-51
- Meant to **practice** and **ask questions**: no new material



## Literature

We'll use several sources, in particular research papers; announced on the website.

First couple lectures, a new (draft) book:



Jan Rutten. The method of coalgebra: exercises in coinduction.

<http://www.cwi.nl/~janr/papers/files-of-papers/course.pdf>

We will also sometimes use:



Bart Jacobs. Introduction to coalgebra, version 2.0, 2012.

<http://www.cs.ru.nl/B.Jacobs/CLG/JacobsCoalgebraIntro.pdf>

Official version published by Cambridge University Press, 2016.

Second half (mostly): lecture notes



## Assessment

- Two graded assignments: end of October, end of December (separate from the homework in the exercise classes, which is not graded)
- A final exam (January 11th 2019)
- Final grade:  $(H+E)/2$ , where H is the grade given for the homework assignments and E is the grade given for the final exam



# Coalgebra

- Mathematical theory of **state-based systems**



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- Uniform study of many types of systems and models; various kinds of automata, (infinite) data structures such as sequences and trees, probabilistic systems, ...





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- Applications, fundamental insights, new perspectives



## A small bit of history

- Machines and data structures in category theory (70's, 80's)
- Behavioural equivalence of processes (Milner & Park, 80's)
- Processes and non-wellfounded set theory (Peter Aczel (late 80's))
- Universal coalgebra (Jan Rutten and others – from 90's on)
- By the end of the 90's: active and diverse area of research, conference & workshop of it's own



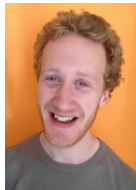
## Coalgebra research today

Active community studying the fundamental theory of coalgebras, as well as applications in/connections to:

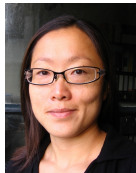
- automata theory
- programming language semantics
- concurrency theory
- probabilistic systems
- modal logic
- theorem proving
- type theory
- ...



## Coalgebra in Nijmegen



and people who now work elsewhere:



# What will you learn

In this course:

- basic theory — modelling systems as coalgebras



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- applying the coalgebraic notions of behavioural equivalence and coinduction



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- the interplay between algebra and coalgebra, induction and coinduction





# What will you learn

In this course:

- basic theory — modelling systems as coalgebras
- applying the coalgebraic notions of behavioural equivalence and coinduction
- the interplay between algebra and coalgebra, induction and coinduction
- using coalgebraic/coinductive proof techniques;
  - *highlight: a recent breakthrough algorithm for equivalence of non-deterministic automata*



# Overview

## Basic theory of coalgebras and coinduction (first quarter)

- streams
- categories, coalgebras, bisimulation
- automata

## Algebra and coalgebra (second quarter)

- algebras, induction
- monads
- distributive laws

## Bisimulation and coinduction: advanced topics (second quarter)

- lattices and fixed point theory
- minimization
- proof techniques and algorithms, applications

